HOMEWORK 11 FOR MATH 2250 - MODEL SOLUTION

Every problem will be worth two points.

(1) Use Newton's method to estimate the one real solution of $x^3 + 3x + 1 = 0$. Start with $x_0 = 0$ and then find x_2.

$$f(x) = x^3 + 3x + 1 \Rightarrow f'(x) = 3x^2 + 3.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 + 3x_n + 1}{3x_n^2 + 3}.$$

$$x_1 = x_0 - \frac{x_0^3 + 3x_0 + 1}{3x_0^2 + 3} = 0 - \frac{0^3 + 3 \cdot 0 + 1}{3 \cdot 0^2 + 3} = -\frac{1}{3}.$$

$$x_2 = x_1 - \frac{x_1^3 + 3x_1 + 1}{3x_1^2 + 3} = -\frac{1}{3} - \frac{\left(-\frac{1}{3}\right)^3 + 3 \cdot \left(-\frac{1}{3}\right) + 1}{3 \cdot \left(-\frac{1}{3}\right)^2 + 3} = -\frac{29}{90} \approx -0.3222222.$$

(2) The curve $y = \tan x$ crosses the line $y = 2x$ between $x = 0$ and $x = \pi/2$. Use Newton's method to find where.

$y = \tan x$ crosses $y = 2x$ when $\tan x = 2x$ or equivalently, $\tan x - 2x = 0$.
Define $f(x) = \tan x - 2x$.

$$f'(x) = \sec^2 - 2.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\tan x_n - 2x_n}{\sec^2 x_n - 2}.$$

Set $x_0 = 1$.

$$x_1 = x_0 - \frac{\tan x_0 - 2x_0}{\sec^2 x_0 - 2} = 1 - \frac{\tan 1 - 2 \cdot 1}{\sec^2 1 - 2} \approx 1.31047803009.$$

$$x_2 = x_1 - \frac{\tan x_1 - 2x_1}{\sec^2 x_1 - 2} \approx 1.2239290965.$$

$$x_3 = x_2 - \frac{\tan x_2 - 2x_2}{\sec^2 x_2 - 2} \approx 1.1760509.$$

$$x_4 = x_3 - \frac{\tan x_3 - 2x_3}{\sec^2 x_3 - 2} \approx 1.16592650831.$$

$$x_5 = x_4 - \frac{\tan x_4 - 2x_4}{\sec^2 x_4 - 2} \approx 1.16556163635.$$

$$x_6 = x_5 - \frac{\tan x_5 - 2x_5}{\sec^2 x_5 - 2} \approx 1.16556118521.$$

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(3) Find the definite integral

$$\int 2\cos 2x - 3\sin 3x dx.$$

$$\begin{aligned}\int 2\cos 2x - 3\sin 3x &= \int 2\cos 2x - \int 3\sin 3x dx \\ &= 2\int \cos 2x - 3\int \sin 3x dx \\ &= 2\cdot\frac{1}{2}\sin 2x - 3\cdot\left(-\frac{1}{3}\cos 3x\right) + C \\ &= \sin 2x + \cos 3x + C.\end{aligned}$$

(4) Solve the initial value problem

$$\frac{d^2s}{dt^2} = \frac{3t}{8}, \quad \left.\frac{ds}{dt}\right|_{t=4} = 3, \quad s(4) = 4.$$

$$\frac{d^2s}{dt^2} = \frac{3t}{8} \Rightarrow \frac{ds}{dt} = \frac{3}{16}t^2 + C.$$

$$3 = \left.\frac{ds}{dt}\right|_{t=4} = \frac{3}{16}4^2 + C = 3 + C \Rightarrow C = 0.$$

$$\frac{ds}{dt} = \frac{3}{16}t^2.$$

$$\frac{ds}{dt} = \frac{3}{16}t^2 \Rightarrow s = \frac{1}{16}t^3 + K.$$

$$4 = s(4) = \frac{1}{16}4^3 + K = 4 + K \Rightarrow K = 0.$$

$$s = \frac{1}{16}t^3.$$