Homework 5 Model Solution

Section 14.1.

14.1.9 Let $g(x, y) = \cos(x + 2y)$.

(a) Evaluate $g(2, -1)$.

$$g(2, -1) = \cos(2 + 2(-1)) = \cos 0 = 1$$

(b) Find the domain of g.

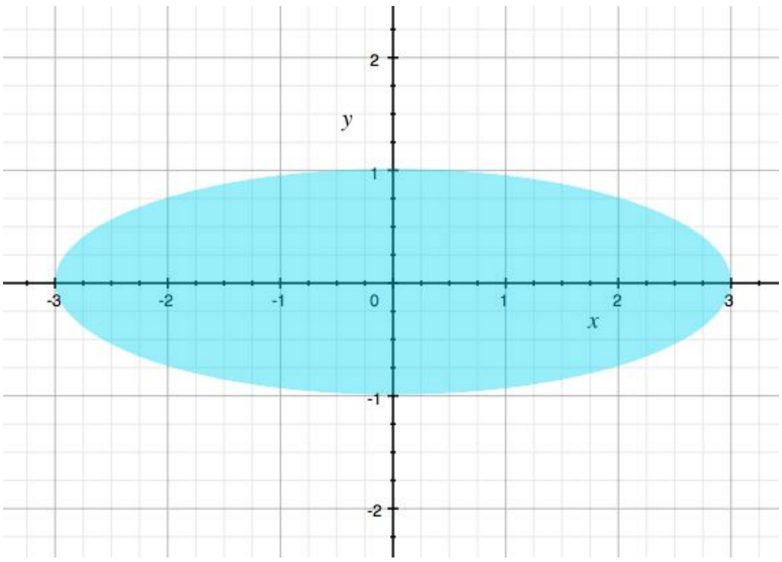
Cosine is defined for all real numbers. So x and y can be arbitrary numbers. Therefore the domain is whole $\mathbb{R}^2$.

(c) Find the range of g.

The range of cosine is $[-1, 1]$. So the range of g is $[-1, 1]$ as well.

14.1.15 Find and sketch the domain of $f(x, y) = \ln(9 - x^2 - 9y^2)$.

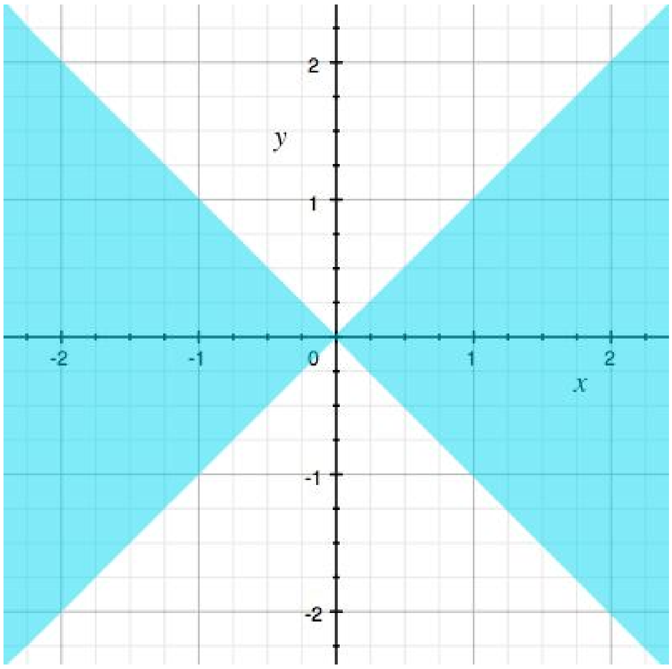
$\ln t$ is defined only if $t > 0$. So $9 - x^2 - 9y^2 > 0$ or $x^2 + 9y^2 < 9$. Therefore the domain is the interior of an ellipse defined by $x^2 + 9y^2 = 9$.



14.1.16 Find and sketch the domain of the function of $f(x, y) = \sqrt{x^2 - y^2}$.

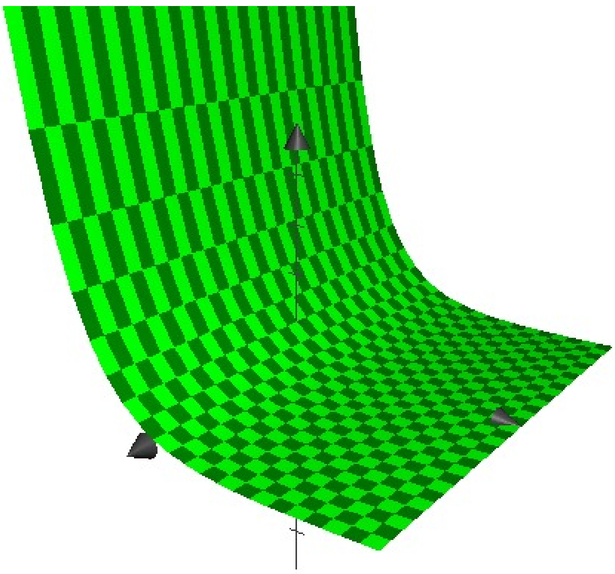
The inside of a square root must be nonnegative. So $f(x, y)$ is defined only if $x^2 - y^2 \geq 0$. In other words, the domain is $x^2 - y^2 \geq 0$.

Note that $x^2 - y^2 = (x + y)(x - y) = 0$. Therefore the boundary is the union of two diagonal lines passing through the origin. The domain does contain the boundaries.



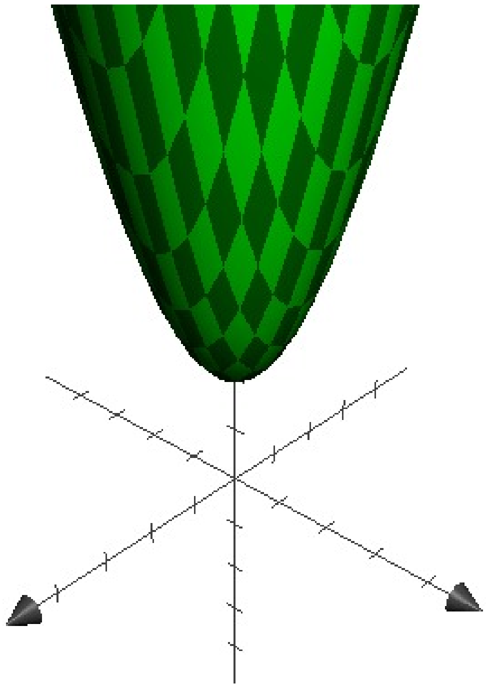
14.1.26 Sketch the graph of $f(x, y) = e^{-y}$.

Because the function $f(x, y)$ does not depends on x, the section of the graph of f by a plane $x = a$ is always the graph of $z = e^{-y}$.



14.1.28 Sketch the graph of $f(x, y) = 1 + 2x^2 + 2y^2$.

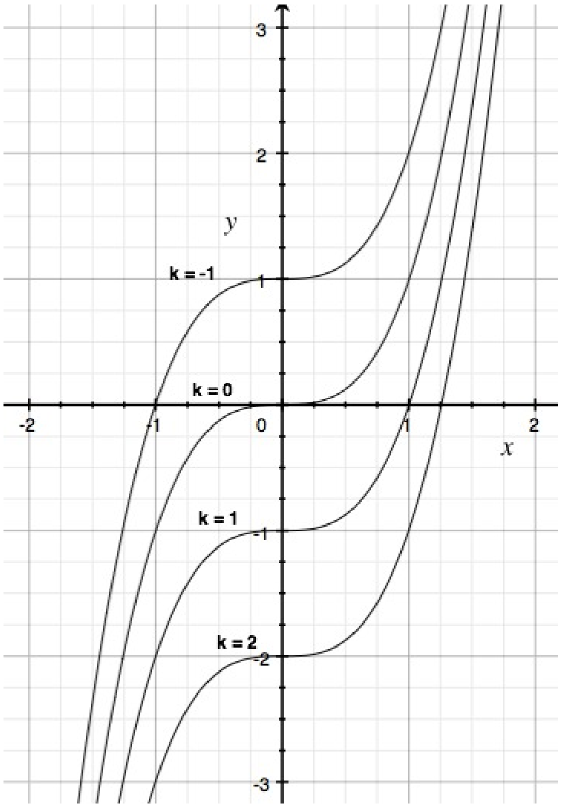
Note that $1 + 2x^2 + 2y^2 = 1 + 2r^2$. So the graph of f is the rotation of the graph $z = 1 + 2r^2$ (which is a parabola) about z-axis.



14.1.44 Draw a contour map of $f(x, y) = x^3 - y$ showing several level curves.

$$x^3 - y = k \Leftrightarrow y = x^3 - k$$

So a level curve is the graph of $y = x^3 - k$.



14.1.50 Draw a contour map of $f(x,y) = \dfrac{y}{x^2+y^2}$ showing several level curves.

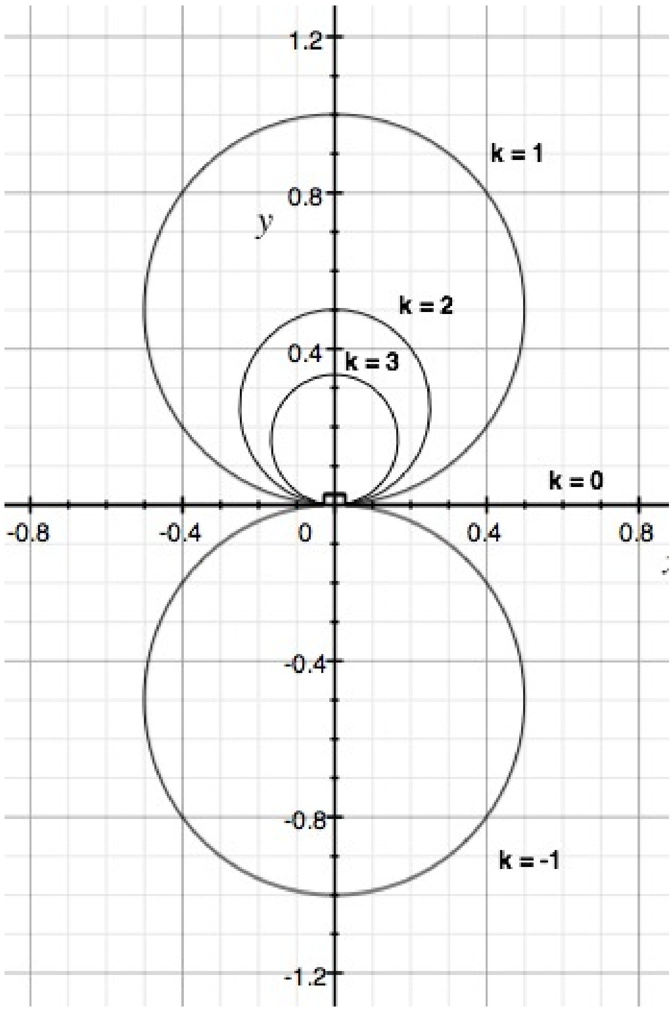
if $k \neq 0$,

$$\frac{y}{x^2+y^2} = k \Leftrightarrow y = k(x^2+y^2) \Leftrightarrow kx^2 + ky^2 - y = 0$$

$$\Leftrightarrow x^2 + y^2 - \frac{1}{k}y = 0 \Leftrightarrow x^2 + \left(y - \frac{1}{2k}\right)^2 = \left(\frac{1}{2k}\right)^2$$

So the level set is a circle of radius $\dfrac{1}{2k}$ and center $(0, \dfrac{1}{2k})$.

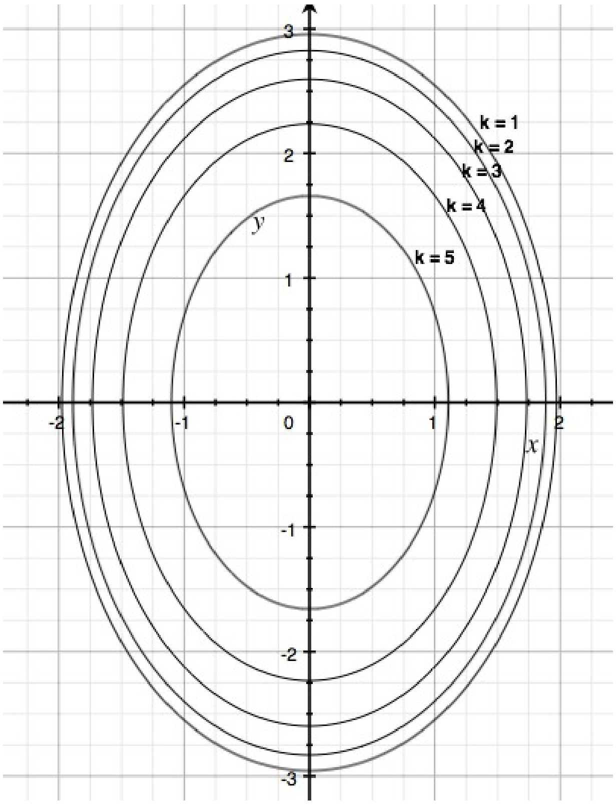
If $k = 0$, $y = 0$ and it is x-axis.



14.1.52 Sketch both a contour map and a graph of $f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$ and compare them.

$$\sqrt{36 - 9x^2 - 4y^2} = k \Leftrightarrow 36 - 9x^2 - 4y^2 = k^2 \Leftrightarrow 9x^2 + 4y^2 = 36 - k^2$$

Therefore a level curve is an ellipse.



The graph looks like a bowl. The graph below is just a part of the whole graph.