Minor correction to "Equations for point configurrations to lie on a rational normal curve"

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In [CGMS18, §4], we are concerned with n-point configurations $\mathbf{p} = (p_1, \ldots, p_n) \in (\mathbb{P}^d)^n$ satisfying the property that every hyperplane $H \subseteq \mathbb{P}^d$ avoids at least two points in the configuration. Let us call such n-point configurations *strongly non-degenerate*. We consider these point configurations because if $\mathbf{p}$ is strongly non-degenerate, then the set of Gale transforms $\widetilde{G}(\mathbf{p})$ is well defined.

In [CGMS18, §4], such n-point configurations are mistakenly called "automorphism-free" point configurations, meaning that if a projective linear transformation fixes the n points, then that has to be the identity. As David Speyer kindly pointed out to us, although an automorphism-free point configuration is strongly non-degenerate, the converse is false.

As a counterexample, consider the two skew lines $\{X_0 = X_1 = 0\}, \{X_2 = X_3 = 0\}$ in $\mathbb{P}^3$ and a configuration of $n \geq 6$ distinct points on these two lines with at least three points on each line. This n-point configuration is strongly non-degenerate, but it is not automorphism-free. Nontrivial automorphisms of $\mathbb{P}^3$ fixing the n-points are

$$\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & \mu \end{pmatrix},$$

for all choices of $\lambda, \mu \in \Bbbk \setminus \{0\}$, $\lambda \neq \mu$, where $\Bbbk$ is our base field.

Therefore, in [CGMS18, §4] the name automorphism-free point configuration has to be replaced with strongly non-degenerate, and Lemma 4.7, which is false, has to be deleted. All the proofs and theorems remain valid, since we never make use of the automorphism-free property, but just of the weaker strongly non-degenerate one.

References

[CGMS18] Alessio Caminata, Noah Giansiracusa, Han-Bom Moon, and Luca Schaffler. Equations for point configurations to lie on a rational normal curve. Adv. Math. 340 (2018), 653–683.