MATH 1207 R04 MIDTERM EXAM 1 SOLUTION

SPRING 2017 - MOON

Name: ______________________________

- Write your answer neatly and show steps.
- Except calculators, any electronic devices including laptops and cell phones are not allowed.

(1) Quick survey.

(a) (1 pt) This class is:

Too easy			Moderate			Too difficult
1	2	3	4	5	6	7

(b) (2 pts) Write any suggestion for improving this class. (For instance, give more examples in class, explain proofs in detail, give more homework, slow down the pace, ...)

(2) (7 pts) Find the average of $3\sin x$ on the interval $[0, \pi]$.

$$\text{Average} = \frac{1}{\pi - 0}\int_0^\pi 3\sin x dx$$

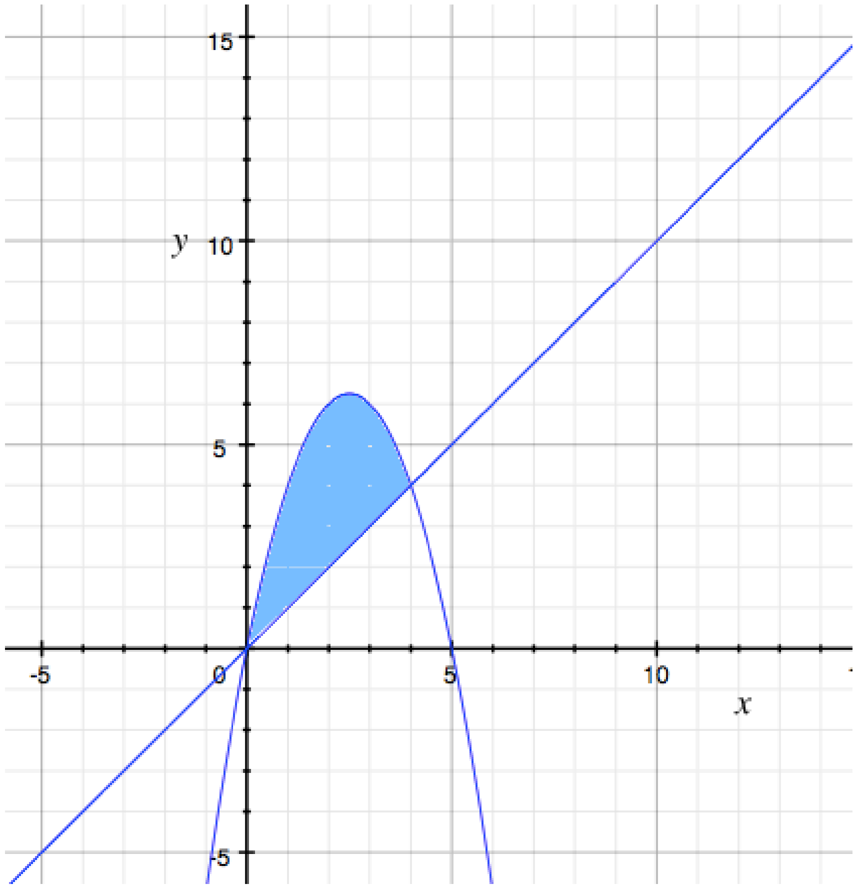
$$\int_0^\pi 3\sin x dx = -3\cos x]_0^\pi = -3\cos\pi - (-3\cos 0) = 6$$

$$\text{Average} = \frac{1}{\pi - 0}\int_0^\pi 3\sin x dx = \frac{6}{\pi}$$

- Writing the formula $\frac{1}{\pi - 0}\int_0^\pi 3\sin x dx$: +3 pts.
- Evaluating the integral $\int_0^\pi 3\sin x dx = 6$: + 3 pts.
- Getting the answer $\frac{6}{\pi}$: +1 pt.

Date: February 17, 2017.

(3) (a) (5 pts) Sketch the region bounded by $y = 5x - x^2$ and $y = x$.



- Sketching two curves: +3 pts.
- Describing two intersection points $(0, 0)$ and $(4, 4)$: + 2 pts.

(b) (7 pts) Find the area of the region in (a).

$$\text{Area} = \int_0^4 5x - x^2 - x dx = \int_0^4 4x - x^2 dx = 2x^2 - \frac{x^3}{3}\Big]_0^4$$

$$= \left(2 \cdot 16 - \frac{64}{3}\right) = \frac{32}{3}$$

- Setting up the appropriate integral $\int_0^4 5x - x^2 - x dx$: 4 pts.
- Getting the answer $\frac{32}{3}$: 7 pts.

(4) (8 pts) Find the volume of the solid generated by rotating the region in the previous problem about x-axis.

Cross section: washer

Inner radius: x, Outer radius: $5x - x^2$

Area of the cross section: $\pi(5x - x^2)^2 - \pi x^2$

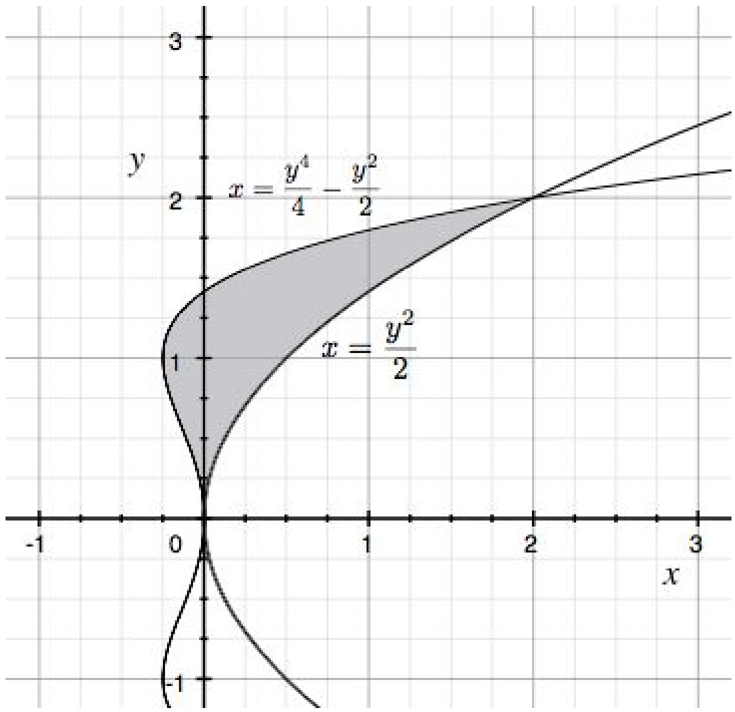
$$\begin{aligned} \text{Volume } &= \int_0^4 \pi(5x - x^2)^2 - \pi x^2 dx = \pi \int_0^4 24x^2 - 10x^3 + x^4 dx \\ &= \pi \left(8x^3 - \frac{10}{4}x^4 + \frac{x^5}{5} \Big]_0^4 \right) = \frac{384\pi}{5} \end{aligned}$$

- Mentioning that the shape of the cross section is a washer: 2 pts.
- Finding the formula for the area of the cross section $\pi(5x - x^2)^2 - \pi x^2$: 5 pts.
- Writing the integral $\int_0^4 \pi(5x - x^2)^2 - \pi x^2 dx$ for the volume: 6 pts.
- Getting the answer $\frac{384\pi}{5}$: 8 pts.

(5) (10 pts) Find the volume of the solid generated by revolving the shaded region bounded by

$$x = \frac{y^4}{4} - \frac{y^2}{2}, \quad x = \frac{y^2}{2}$$

about x-axis.



We will use the shell method.

Radius: y

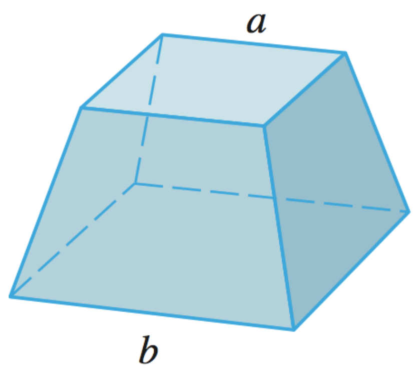
Height: $\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right) = y^2 - \frac{y^4}{4}$

$$\text{Volume} = \int_0^2 2\pi y\left(y^2 - \frac{y^4}{4}\right)dy = 2\pi\int_0^2 y^3 - \frac{y^5}{4}dy$$

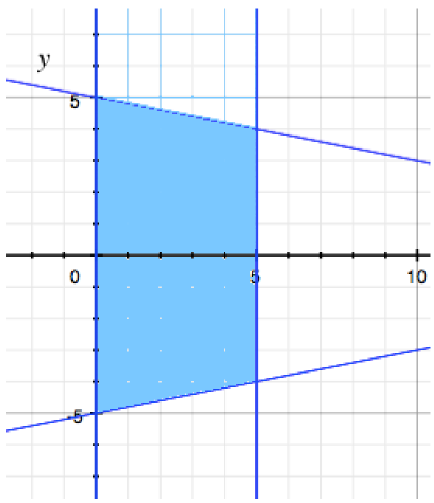
$$= 2\pi\left(\frac{y^4}{4} - \frac{y^6}{24}\Big]_0^2\right) = 2\pi\left(\frac{16}{4} - \frac{64}{24}\right) = \frac{8\pi}{3}$$

- Finding the radius and the height: +2 pts each.
- Stating the integral $\int_0^2 2\pi y\left(y^2 - \frac{y^4}{4}\right)dy$ for the volume: 7 pts.
- Getting the answer $\frac{8\pi}{3}$: 10 pts.

(6) (10 pts) Consider a frustum of a pyramid with square base of side $b = 10$, square top of side $a = 8$, and height $h = 5$. Find the volume of the frustum.



Below is the picture of the frustum which was seen on the side. Let x be the vertical coordinate and suppose that for the base $x = 0$ (so for the top $x = 5$).



The cross section is a square. Suppose that t is the length of the side of the cross section when the vertical coordinate is x. Because the length of the side is linearly decreasing, $t = mx + b$. Furthermore, $t(0) = 10$ and $t(5) = 8$. So $b = 10$ and $m = -2/5$. Thus

$$t = -\frac{2}{5}x + 10$$

and

$$A(x) = \left(-\frac{2}{5}x + 10\right)^2 = \frac{4x^2}{25} - 8x + 100.$$

Therefore

$$\text{Volume } = \int_0^5 \frac{4x^2}{25} - 8x + 100dx = \frac{4x^3}{75} - 4x^2 + 100x\Big]_0^5 = \frac{500}{75} - 100 + 500 = \frac{1220}{3}.$$

- Introducing an appropriate coordinate and mentioning that the cross section is a square: 3 pts.
- Finding the length of the side of the cross section $-\frac{2}{5}x + 10$: 6 pts.

- Setting up the integral $\int_0^5 \frac{4x^2}{25} - 8x + 100dx$ for the volume: 8 pts.
- Getting the answer $\frac{1220}{3}$: 10 pts.