MATH 1100 FINAL SOLUTION

SPRING 2015 - MOON

- Write your answer neatly and show steps.
- Except calculators, any electronic devices including laptops and cell phones are not allowed.
- Do not use the graphing function on your calculator.

(1) Suppose that $A = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -4 \\ 2 & 0 \end{bmatrix}$.

(a) (3 pts) Compute $5A + 3B$.

$$5A + 3B = 5\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} + 3\begin{bmatrix} 7 & -4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 15 & 25 \\ 10 & 15 \end{bmatrix} + \begin{bmatrix} 21 & -12 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 36 & 13 \\ 16 & 15 \end{bmatrix}$$

- Making a mistake to compute an entry: -1 pt each.

(b) (3 pts) Compute AB.

$$AB = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \cdot 7 + 5 \cdot 2 & 3 \cdot (-4) + 5 \cdot 0 \\ 2 \cdot 7 + 3 \cdot 2 & 2 \cdot (-4) + 3 \cdot 0 \end{bmatrix} = \begin{bmatrix} 31 & -12 \\ 20 & -8 \end{bmatrix}$$

- Making a mistake to compute an entry: -1 pt each.

Date: May 7, 2015.

(c) (5 pts) Find A^{-1}.

$$\left[\begin{array}{cccc} 3 & 5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_1 \to R_1} \left[\begin{array}{cccc} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1 \to R_2} \left[\begin{array}{cccc} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$$\xrightarrow{-3R_2 \to R_2} \left[\begin{array}{cccc} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 2 & -3 \end{array}\right] \xrightarrow{R_1 - \frac{5}{3}R_2 \to R_1} \left[\begin{array}{cccc} 1 & 0 & -3 & 5 \\ 0 & 1 & 2 & -3 \end{array}\right]$$

Therefore $A^{-1} \left[\begin{array}{cc} -3 & 5 \\ 2 & -3 \end{array}\right]$.

- Setting the matrix $\left[\begin{array}{cccc} 3 & 5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$: 2 pts.
- Getting the answer $A^{-1} \left[\begin{array}{cc} -3 & 5 \\ 2 & -3 \end{array}\right]$: 5 pts.
- Making a computational error during performing row transformations: -1 pt each.

(2) A furniture manufacturing company manufactures dining-room tables and chairs. The relevant manufacturing data are given in the table below.

Department	Labor-Hours per Unit		Maximum Labor-Hours Available per Day
	Table	Chair	
Assembly	8	2	400
Finishing	2	1	120
Revenue per unit	\$90	\$25	

They want to find the maximum revenue per day.

(a) (2 pts) Identify the variables and write the constraints as inequalities.

$x =$ number of produced tables in a day, $y =$ number of produced chairs in a day

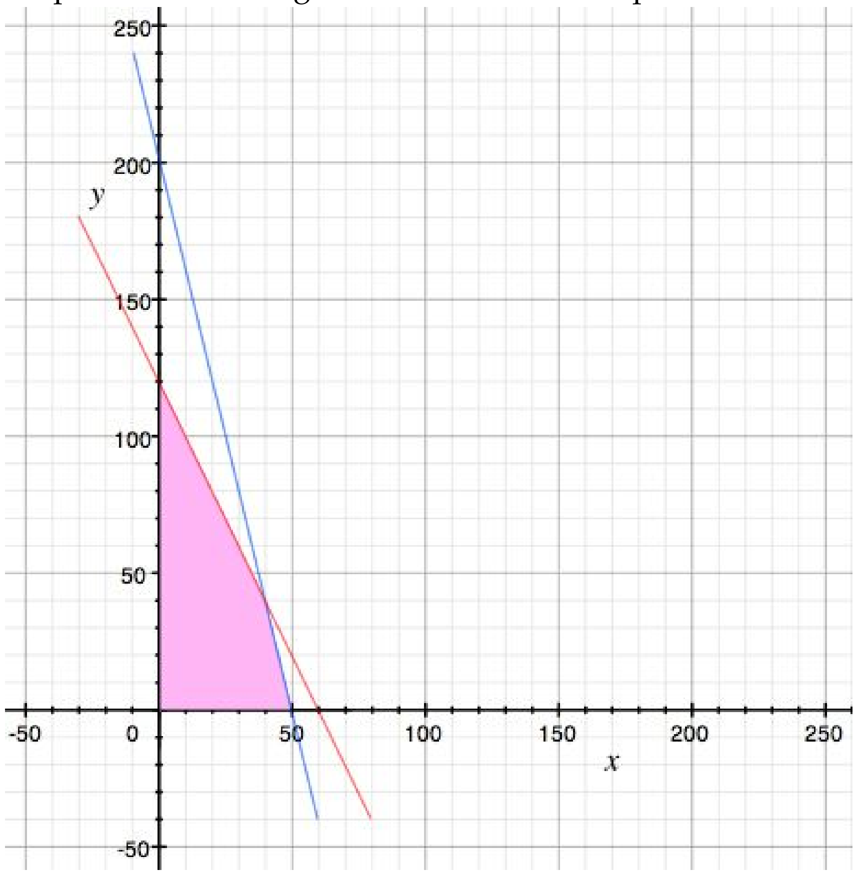
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 8x + 2y \leq 400 \\ 2x + y \leq 120 \end{cases}$$

- Indicating the meaning of two variables correctly: 1 pt.
- Finding a list of inequalities: 1 pt.

(b) (1 pt) Write the objective function.

$$z = 90x + 25y$$

(c) (5 pts) Graph the feasible region and find all corner points.



$$8x+2y=400, 2x+y=120 \Rightarrow 8x+2y=400, y=-2x+120 \Rightarrow 8x+2(-2x+120)=400$$
$$\Rightarrow 4x+240=400 \Rightarrow x=40, y=-2\cdot 40+120=40$$

Corner points: $(0,0), (50,0), (0,120), (40,40)$

- Sketching graph: 3 pts.
- Finding all corner points $(0,0), (50,0), (0,120), (40,40)$: 5 pts.

(d) (2 pts) Find the maximum profit and determine how many tables and chairs should be manufactured each day to maximize the profit.

Corner points	profit
$(0,0)$	0
$(50,0)$	4500
$(0,120)$	3000
$(40,40)$	4600

Maximum profit: $\$4,600$
It occurs when 40 tables and 40 chairs are made.

- Comparing the objective function at all corner points and finding the maximum $\$4,600$: 1 pt.
- Indicating the numbers of tables (40) and chairs (40): 2 pts.

(3) (9 pts) Use the *Gauss-Jordan method* to solve

$$\begin{aligned} x + 2y - 7z &= -2 \\ -2x - 5y + 2z &= 1 \\ 3x + 5y + 4z &= -9. \end{aligned}$$

Write all steps on your computation.

$$\begin{bmatrix} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{bmatrix} \xrightarrow{2R_1+R_2\to R_2} \begin{bmatrix} 1 & 2 & -7 & -2 \\ 0 & -1 & -12 & -3 \\ 3 & 5 & 4 & -9 \end{bmatrix}$$

$$\xrightarrow{-3R_1+R_3\to R_3} \begin{bmatrix} 1 & 2 & -7 & -2 \\ 0 & -1 & -12 & -3 \\ 0 & -1 & 25 & -3 \end{bmatrix} \xrightarrow{-R_2\to R_2} \begin{bmatrix} 1 & 2 & -7 & -2 \\ 0 & 1 & 12 & 3 \\ 0 & -1 & 25 & -3 \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & -31 & -8 \\ 0 & 1 & 12 & 3 \\ 0 & -1 & 25 & -3 \end{bmatrix} \xrightarrow{R_2+R_3\to R_3} \begin{bmatrix} 1 & 0 & -31 & -8 \\ 0 & 1 & 12 & 3 \\ 0 & 0 & 37 & 0 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{37}R_3\to R_3} \begin{bmatrix} 1 & 0 & -31 & -8 \\ 0 & 1 & 12 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{31R_3+R_1\to R_1} \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 12 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

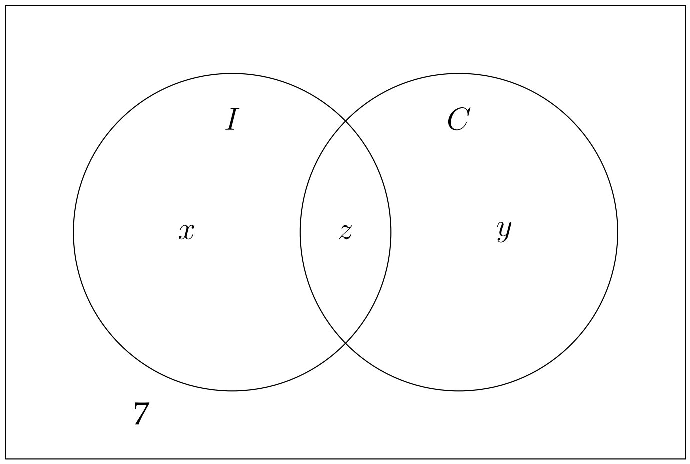
$$\xrightarrow{-12R_3+R_2\to R_2} \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow x = -8,\ y = 3,\ z = 0$$

- Writing the corresponding matrix $\begin{bmatrix} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{bmatrix}$: 2 pts.
- Obtaining the row echelon matrix $\begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$: 8 pts.
- Getting the answer $x = -8\ y = 3,\ z = 0$: 9 pts.
- Making a mistake for a row transformation: -2 pts each.

(4) In the last semester, 52 of students who took Dr. Moon's mathematics class completed the course evaluation. 38 students answered that his class was interesting, and 42 students answered that it was intellectually challenging, and 7 students answered that it was not interesting nor intellectually challenging.

(a) (4 pts) Find the number of students who responded that his class was interesting and intellectually challenging.



Suppose that I is the set of students who responded that Dr. Moon's class was interesting, and C is the set of students who responded that his class was intellectually challenging. Then from the assumption, $n(I) = 38$, $n(C) = 42$, and $n(I' \cap C') = 7$.

$$n(I \cup C) = 52 - 7 = 45$$

$$n(I \cup C) = n(I) + n(C) - n(I \cap C) \Rightarrow 45 = 38 + 42 - n(I \cap C) \Rightarrow n(I \cap C) = 35$$

- Indicating that $n(I \cap C)$ is what we need to find: 2 pts.
- Getting the answer 35: 4 pts.

(b) (4 pts) Find the number of students who responded that his class was interesting but not intellectually challenging.

$$n(I) = x + z \Rightarrow 38 = x + 35 \Rightarrow x = 3$$

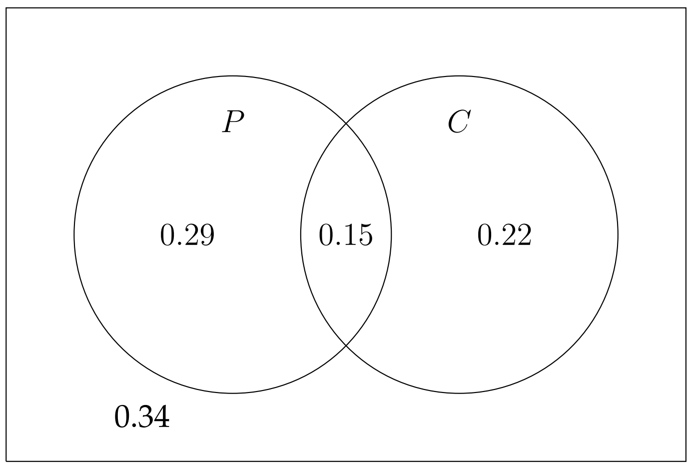
(5) A store sells printers and copiers. Let P be the event that "a customer buys a printer", and let C be the event that "a customer buys a copier". Suppose that $P(P) = 0.51$, $P(C) = 0.37$, $P(P \cap C) = 0.22$.

(a) (3 pts) Find $P(P \cup C)$.

$$P(P \cup C) = P(P) + P(C) - P(P \cap C) = 0.51 + 0.37 - 0.22 = 0.66$$

- Stating $P(P \cup C) = P(P) + P(C) - P(P \cap C)$: 1 pt.
- Getting the answer 0.66: 3 pts.

(b) (3 pts) Find $P(P' \cup C)$.



$$P(P' \cup C) = 0.22 + 0.15 + 0.34 = 0.71$$

- Sketching Venn-Diagram with and the probability distribution: 1 pt.
- Getting the answer 0.71: 3 pts.

(c) (2 pts) Write the event "a customer buys neither machine" using $\cap$, $\cup$, or $'$ as necessary.

$$P' \cap C'$$

(d) (3 pts) Find the probability of the event in (c).

$$P(P' \cap C') = 0.34$$

(6) On a typical January day in Manhattan, the probability of snow is 0.10, the probability of a traffic jam is 0.80, and the probability of snow or a traffic jam (or both) is 0.82.

(a) (3 pts) Find the probability that it snows and a traffic jam occurs.
Suppose that S is the event that it snows and T is the event that a traffic jam occurs. Then from the assumption, $P(S) = 0.1$, $P(T) = 0.8$ and $P(S \cup T) = 0.82$.

$$P(S \cup T) = P(S) + P(T) - P(S \cap T) \Rightarrow 0.82 = 0.1 + 0.8 - P(S \cap T) \Rightarrow P(S \cap T) = 0.08$$

- Indicating that $P(S \cap T)$ is the probability we want to find: 1 pt.
- Getting the answer 0.08: 3 pts.

(b) (3 pts) Find the probability that a traffic jam occurs on a snow day.

$$P(T|S) = \frac{P(S \cap T)}{P(S)} = \frac{0.08}{0.1} = 0.8$$

- Indicating that $P(T|S)$ is the probability we want to find: 1 pt.
- Describing $P(T|S)$ as a fraction $\frac{P(S \cap T)}{P(S)}$: 2 pts.
- Obtaining the answer 0.8: 3 pts.

(c) (3 pts) Are two events "it snows" and "a traffic jam occurs" independent? Explain your answer.

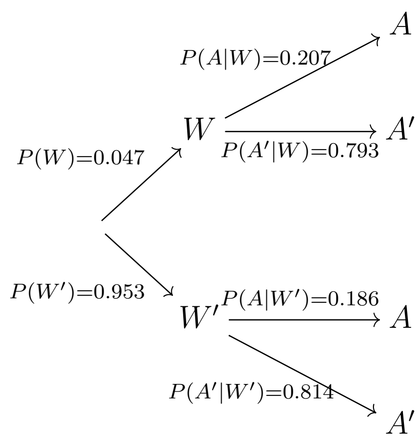
Because $P(T|S) = 0.8 = P(T)$, these two events are independent.

- Getting the correct answer: 1 pt.
- Explaining the reason well: 3 pts.

(7) The Harvard School of Public Health completed a study on alcohol consumption on college campuses in 2001. They concluded that 20.7% of women attending all-women colleges abstained from alcohol, compared to 18.6% of women attending coeducational colleges. Approximately 4.7% of women college students attend all-women schools.

(a) (5 pts) What is the probability that a randomly selected female student abstains from alcohol?

Suppose that W is the event that a randomly selected female student attends all-women school, A is the event that a chosen student abstained from alcohol.



$$P(A) = P(A \cap W) + P(A \cap W') = P(W)P(A|W) + P(W')P(A|W')$$

$$= 0.047 \cdot 0.207 + 0.953 \cdot 0.186 \approx 0.1870$$

- Sketching the tree diagram correctly: 2 pts.
- Indicating the probability $P(A)$ we want to find: 3 pts.
- Getting the answer 0.1870: 5 pts.

(b) (5 pts) If a randomly selected female student abstains from alcohol, what is the probability she attends a coeducational college?

$$P(W'|A) = \frac{P(W' \cap A)}{P(A)} = \frac{0.953 \cdot 0.186}{0.047 \cdot 0.207 + 0.953 \cdot 0.186} \approx 0.9480$$

- Indicating the probability $P(W'|A)$ we want to find: 2 pts.
- Getting the answer 0.9480: 5 pts.

(8) The admission committee of 3 members is to be selected from 3 departments in a college. There are 7 professors in Mathematics department, 5 in Physics department, and 6 in Chemistry department.

(a) (3 pts) How many different committees are possible?

There are $7 + 5 + 6 = 18$ professors. Because we need to choose 3 of them without regarding the order,

$$C(18, 3) = \frac{18!}{15!3!} = 816.$$

- Finding the correct formula $C(18, 3)$: 2 pts.
- Getting the answer 816: 3 pts.

(b) (3 pts) How many different committees with exactly 2 mathematics professors are possible?

We need to choose 2 mathematics professors and one non-mathematics professor from remaining 11 professors.

$$C(7, 2) \times C(11, 1) = \frac{7!}{5!2!} \times \frac{11!}{10!1!} = 231$$

- Finding the correct formula $C(7, 2) \times C(11, 1)$: 2 pts.
- Getting the answer 231: 3 pts.

(c) (3 pts) How many different ways to select a chair, a vice chair and a secretary are possible?

Now the order of choices does matter.

$$P(18, 3) = 18 * 17 * 16 = 4896$$

- Finding the correct formula $P(18, 3)$: 2 pts.
- Getting the answer 4896: 3 pts.

(d) (3 pts) If the committee must have at least one professor from each department, how many different committees are possible?

The only way to make it is to choose one from mathematics department, one from physics, and one from chemistry department. Therefore the number of ways is $7 \times 5 \times 6 = 210$.

- If the idea of computation is wrong, you can't get any credit even though the answer is correct.

(9) During 2014-2015 season, the New York Knicks basketball team won 10 home games and lost 31 home games. A film director Spike Lee is a famous fan of Knicks. Suppose that he attended 12 games that season randomly.

(a) (4 pts) Find the probability that the team lose all of the 12 games.

The number of ways to choose 12 games from 41 home games is $C(41, 12)$. And the number of ways to choose 12 games from 31 lost home games is $C(31, 12)$. So the probability is:

$$\frac{C(31,12)}{C(41,12)} = \frac{141120525}{7898654920} \approx 0.0179$$

- Getting the formula $\frac{C(31,12)}{C(41,12)}$: 3 pts.
- Obtaining the answer 0.0179: 4 pts.

(b) (4 pts) Find the probability that the team won 3 of the games and lose 9 games.

$$\frac{C(10,3) \cdot C(31,9)}{C(41,12)} = \frac{120 \times 20160075}{7898654920} \approx 0.3063$$

- Getting the formula $\frac{C(10,3) \cdot C(31,9)}{C(41,12)}$: 3 pts.
- Obtaining the answer 0.3063: 4 pts.

(10) According to a survey by Javelin Strategy and Research, 1 out of 6 adults in Arizona were victims of identity theft. Suppose that 20 adults are randomly selected from Arizona.

(a) (4 pts) Find the probability that exactly two adults were victims of identity theft.

The probability that a chosen adult was a victim of identity theft is $\frac{1}{6}$.

$$P(2\text{ victims}) = C(20,2)\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^{20-2} \approx 0.1982$$

- Getting the formula $C(20,2)\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^{20-2}$: 3 pts.
- Obtaining the answer 0.1982: 4 pts.

(b) (4 pts) Find the probability that at most two adults were victims of identity theft.

$$P(\text{at most 2 victims}) = P(0\text{ victim}) + P(1\text{ vitcim}) + P(2\text{ victims})$$

$$= C(20,0)\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^{20-0} + C(20,1)\left(\frac{1}{6}\right)^1\left(1-\frac{1}{6}\right)^{20-1}$$

$$+C(20,2)\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^{20-2} \approx 0.3287$$

- Getting the formula $C(20,0)\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^{20-0} + C(20,1)\left(\frac{1}{6}\right)^1\left(1-\frac{1}{6}\right)^{20-1} + C(20,2)\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^{20-2}$: 3 pts.
- Obtaining the answer 0.3287: 4 pts.

(c) (4 pts) Find the probability that at least one adult was victims of identity theft.

$$P(\text{at least one victim}) = 1 - P(0\text{ victim})$$

$$= 1 - C(20,0)\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^{20-0} \approx 0.9739$$

- Having the idea $P(\text{at least one victim}) = 1 - P(0\text{ victim})$: 2 pts.
- Getting the formula $1 - C(20,0)\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^{20-0}$: 3 pts.
- Obtaining the answer 0.9739: 4 pts.