# Homework 10 Solution

Section 8.1.

8.1.16. In a club with 16 members, how many ways can a slate of 3 officers consisting of president, vice-president, and secretary/treasurer be chosen?

$$P(16,3) = 16 \times 15 \times 14 = 3360$$

8.1.24. A child has a set of differently shaped plastic objects. There are 3 pyramids, 4 cubes, and 7 spheres.

(a) In how many ways can she arrange the objects in a row if each is a different color?

There are $3+4+7 = 14$ different objects. So the number is $14! = 87,178,291,200$.

(b) How many arrangements are possible if objects of the same shape must be grouped together and each object is a different color?

There are three groups, so there are 3! ways to arrange them. And there are 3! ways to arrange pyramids, 4! ways to arrange cubes, and 7! ways to arrange spheres.

$$3! \times 3! \times 4! \times 7! = 4,354,560$$

(c) In how many distinguishable ways can the objects be arranged in a row if objects of the same shape are also the same color but need not be grouped together?

We have to remove the effect of the ordering of objects in each group. So the number of ways is

$$\frac{14!}{3!4!7!} = 120,120.$$

(d) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected does not matter and each object is a different color?

$$3 \times 4 \times 7 = 84$$

(e) In how many ways can you select 3 objects, one of each shape, if the order in which the objects are selected matters and each object is a different color?

We have to multiply the order of three choices in (d). So the number is $84 \times 3! = 504$.

8.1.30. An automobile manufacturer produces 8 models, each available in 7 different exterior colors, with 4 different upholstery fabrics and 5 interior colors. How many varieties of automobile are available?

$$8 \times 7 \times 4 \times 5 = 1120$$

8.1.36. At an annual college science conference, student presentations are scheduled one after another in the afternoon session. This year, 5 students are presenting in biology, 5 students are presenting in chemistry, and 2 students are presenting in physics.

(a) In how many ways can the presentations be scheduled?
There are $5 + 5 + 2 = 12$ presentations.

$$12! = 479,001,600$$

(b) In how many ways can the presentations be scheduled so that each subject is grouped together?
First of all, we have to make the order of three subjects. There are 3! ways to do it. Then there are 5! ways to arrange biology presentations, another 5! ways to arrange chemistry presentations, and 2! ways to arrange physics presentations.

$$3! \times 5! \times 5! \times 2! = 172,800$$

(c) In how many ways can the presentations be scheduled if the conference must begin and end with a physics presentation?
We have to choose the order of two physics presentations (2! ways to do it) and the order of 10 remaining presentations.

$$2! \times 10! = 7,257,600$$

8.1.44. How many 7-digit telephone numbers are possible if the first digit cannot be zero and

(a) only odd digits may be used?
For each digit, we can use one of 1, 3, 5, 7, and 9. So the number of possible telephone numbers is

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7 = 78,125.$$

(b) the telephone number must be a multiple of 10 (that is, it must end in zero)?
There are 9 choices for the first digit, because 0 cannot be there. The last digit has to be 0, so there is only one possibility. For the remaining 5 digits, we may choose one of 10 numbers freely.

$$9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 1 = 9 \times 10^5 = 900,000$$

(c) the telephone number must be a must be a multiple of 100?
Now the last two digits must be zero.

$$9 \times 10 \times 10 \times 10 \times 10 \times 1 \times 1 = 9 \times 10^4 = 90,000$$

(d) the first 3 digits are 481?
We have to choose remaining 4 digits.

$$10 \times 10 \times 10 \times 10 = 10^4 = 10,000$$

(e) no repetitions are allowed?
For the first digit, there are 9 choices. For the second digit, there are 9 possibilities because we cannot use the number we've used for the first digit. The third digit has 8 possibilities, and so on. Therefore the number of possibilities is

$$9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 9P(9,6) = 544,320.$$