# Homework 11 Solution

Section 8.2 $\sim$ 8.3.

8.2.14. An economics club has 31 members.

(a) If a committee of 4 is to be selected, in how many ways can the selection be made?

$$C(31, 4) = \frac{31!}{27!4!} = 31,465$$

(b) In how many ways can a committee of at least 1 and at most 3 be selected?

$$C(31, 1) + C(31, 2) + C(31, 3) = \frac{31!}{30!1!} + \frac{31!}{29!2!} + \frac{31!}{28!3!} = 4991$$

8.2.18. In a club with 9 male and 11 female members, how many committees of 5 members can be selected that have

(a) at least 4 women?

There are two ways: 4 women and a man, and 5 women.

$$C(11, 4) \times C(9, 1) + C(11, 5) = \frac{11!}{7!4!} \cdot \frac{9!}{8!1!} + \frac{11!}{6!5!} = 3432$$

(b) no more than 2 men?

There are three cases: 2 men and 3 women, 1 man and 4 women, and 5 women.

$$C(9, 2) \times C(11, 3) + C(9, 1) \times C(11, 4) + C(11, 5) = \frac{9!}{7!2!} \cdot \frac{11!}{8!3!} + 3432 = 9372$$

8.2.24. In how many ways can 5 out of 9 plants be arranged in a row on a windowsill?

When we arrange some objects in a row, the order does matter. So

$$P(9, 5) = \frac{9!}{(9-5)!} = 15,120.$$

8.2.32. A group of 9 workers decides to send a delegation of 3 to their supervisor to discuss their grievances.

(a) How many delegations are possible?

$$C(9, 3) = \frac{9!}{6!3!} = 84$$

(b) If it is decided that a particular worker must be in the delegation, how many different delegations are possible?

It is sufficient to choose the other two workers from 8 remaining workers.

$$C(8,2) = \frac{8!}{6!2!} = 28$$

(c) If there are 4 women and 5 men in the group, how many delegations would include at least 1 woman?

We must subtract the number of delegations consisting of men only.

$$C(9,3) - C(5,3) = 84 - \frac{5!}{2!3!} = 74$$

8.2.36. In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 3 barley plants, and 2 rye plants. She wishes to select 4 plants at random.

(a) In how many ways can this be done?

There are $6 + 3 + 2 = 11$ plants.

$$C(11,4) = \frac{11!}{7!4!} = 330$$

(b) In how many ways can this be done if exactly 2 wheat plants must be included?

We must choose 2 of 6 wheat plants and 2 of 5 other plants.

$$C(6,2) \times C(5,2) = \frac{6!}{4!2!} \cdot \frac{5!}{3!2!} = 150$$

8.2.42. In poker, a flush consists of 5 cards with the same suit, such as 5 diamonds.

(a) Find the number of ways of getting a flush consisting of cards with values from 5 to 10 by listing all the possibilities.

For hearts, there are precisely 6 possibilities:

5, 6, 7, 8, 9
5, 6, 7, 8, 10
5, 6, 7, 9, 10
5, 6, 8, 9, 10
5, 7, 8, 9, 10
6, 7, 8, 9, 10.

There are four suits. So the number is $6 \times 4 = 24$.

(b) Find the number of ways to getting a flush consisting of cards with values from 5 to 10 by using combinations.

First of all, we must choose a suit and then we have to choose 5 of 6 numbers.

$$4 \times C(6,5) = 4 \cdot \frac{6!}{1!5!} = 24$$

8.3.2. A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains all yellow apples.

Suppose that $Y$ is the event that all of the sample are yellow apples. The number of ways to choose a sample of 3 apples is $C(11,3) = 165$. The number of ways to choose three yellow apples is $C(4,3) = 4$. Therefore

$$P(Y) = \frac{n(Y)}{n(S)} = \frac{4}{165} \approx 0.0242.$$

8.3.4. A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains more red than yellow apples.

Suppose that $R$ is the event that there are more red apples than yellow apples in the sample. There are two cases: 3 red apples or 2 red apples and one yellow apple. The number of ways to choose is:

$$n(R) = C(7,3) + C(7,2) \times C(4,1) = 35 + 84 = 119$$

Therefore the probability is

$$P(R) = \frac{n(R)}{n(S)} = \frac{119}{165} \approx 0.7212.$$

8.3.42. Heidi Shadix and Boys Shepherd are among 9 representatives making presentations at the annual sales meeting. The presentations are randomly ordered. Find the probability that Heidi is the first presenter and Boyd is the last presenter.

The number of ways to choose the order of 9 presentations is $P(9,9) = 9! = 362,880$. If Heidi is the first presenter and Boyd is the last one, then it is sufficient to choose the order of the remaining 7 presenters. The number of ways is $P(7,7) = 7! = 5040$. Therefore the probability is

$$\frac{5040}{362880} = \frac{1}{72} \approx 0.0139.$$

8.3.66. In the previous section, we found the number of ways to pick 6 different numbers from 1 to 99 in a state lottery. Assuming order is unimportant, what is the probability of picking all 6 numbers correctly to win the big prize?

The number of ways to choose 6 numbers from 1 to 99 is $C(99,6) = 1,120,529,256$. There is only one way to win. Therefore the probability to win is

$$\frac{1}{1120529256} \approx 8.924 \times 10^{-10}.$$

8.3.72. A controversy arose in 1992 over the Teen Talk Barbie doll, each of which was programmed with four sayings randomly picked from a set of 270 sayings. The controversy ways over the saying, "Math class is tough," which some felt gave a negative message toward girls doing well in math. In an interview with *Science*, a spokeswoman for Mattel, the makers of Barbie, said that "There's a less than 1 % chance you're going to get a doll that says math class is tough." Is this figure correct? If not, give the correct figure.

The number of ways to choose 4 sayings from 270 is $C(270,4) = 216,546,345$. To count the number of ways to choose 4 sayings including "math class is tough", we need to count the number of ways to choose 3 sayings from the remaining 269 sayings. The number of ways is $C(269,3) = 3,208,094$. Therefore the probability is

$$\frac{3208094}{216546345} \approx 0.0148.$$

8.3.74. During the 2009 season, the Washington Nationals baseball team won 59 games and lost 103 games. Season ticket holder Stephen Krupin reported in an interview that he watched the team lose all 19 games that he attended that season. The interviewer speculated that this must be a record for bad luck.

(a) Based on the full 2009 season record, calculate the probability that a person would attend 19 Washington Nationals games and the Nationals would lose all 19 games.

The number of ways to choose 19 games from $59 + 103 = 162$ games is $C(162,19) = 2,622,702,479,572,467,406,346,400$. And the number of ways to choose 19 games from 103 lost games is $C(103,19) = 245,631,370,596,035,312,350$. So the probability is

$$\frac{245631370596035312350}{2622702479572467406346400} \approx 9.3656 \times 10^{-5}.$$

(b) However, Mr. Krupin only attended home games. The Nationals has 33 wins and 48 losses at home in 2009. Calculate the probability that a person would attend 19 Washington Nationals home games and the Nationals would lose all 19 games.

The number of ways to choose 19 games from $33 + 48 = 81$ home games is $C(81,19) = 1,514,334,254,464,231,200$. The number of ways to choose 19 games form 48 lost home games is $C(48,19) = 11,541,847,896,480$. Therefore the probability is

$$\frac{11541847896480}{1514334254464231200} \approx 7.6217 \times 10^{-6}.$$