Homework 2 Solution

Section 2.2 $\sim$ 2.4

2.2.28. Use the Gauss-Jordan method to solve the system of linear equations

$$\begin{aligned} x - z &= -3 \\ y + z &= 9 \\ -2x + 3y + 5z &= 33. \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ -2 & 3 & 5 & 33 \end{array}\right] \xrightarrow{2R_1+R_3\to R_3} \left[\begin{array}{cccc} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 0 & 3 & 3 & 27 \end{array}\right]$$

$$\xrightarrow{-3R_2+R_3\to R_3} \left[\begin{array}{cccc} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{aligned} x - z &= -3 \\ y + z &= 9 \\ 0 &= 0 \end{aligned}$$

From $y + z = 9$, we have $y = -z + 9$. Also from $x - z = -3$, $x = z - 3$. Therefore the general solution is given by:

$$(x, y, z) = (z - 3, -z + 9, z).$$

2.2.30. Use the Gauss-Jordan method to solve the system of linear equations

$$\begin{aligned} x + 2y - 7z &= -2 \\ -2x - 5y + 2z &= 1 \\ 3x + 5y + 4z &= -9. \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{array}\right] \xrightarrow{2R_1+R_2\to R_2} \left[\begin{array}{cccc} 1 & 2 & -7 & -2 \\ 0 & -1 & -12 & -3 \\ 3 & 5 & 4 & -9 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_3\to R_3} \left[\begin{array}{cccc} 1 & 2 & -7 & -2 \\ 0 & -1 & -12 & -3 \\ 0 & -1 & 25 & -3 \end{array}\right] \xrightarrow{-R_2\to R_2} \left[\begin{array}{cccc} 1 & 2 & -7 & -2 \\ 0 & 1 & 12 & 3 \\ 0 & -1 & 25 & -3 \end{array}\right]$$

$$\xrightarrow{R_2+R_3\to R_3} \left[\begin{array}{cccc} 1 & 2 & -7 & -2 \\ 0 & 1 & 12 & 3 \\ 0 & 0 & 37 & 0 \end{array}\right] \xrightarrow{-2R_2+R_1\to R_1} \left[\begin{array}{cccc} 1 & 0 & -31 & -8 \\ 0 & 1 & 12 & 3 \\ 0 & 0 & 37 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{37}R_3\to R_3}\begin{bmatrix}1&0&-31&-8\\0&1&12&3\\0&0&1&0\end{bmatrix}\xrightarrow{31R_3+R_1\to R_1}\begin{bmatrix}1&0&0&-8\\0&1&12&3\\0&0&1&0\end{bmatrix}$$

$$\xrightarrow{-12R_3+R_2\to R_2}\begin{bmatrix}1&0&0&-8\\0&1&0&3\\0&0&1&0\end{bmatrix}\Leftrightarrow\begin{array}{rcl}x&=&-8\\y&=&3\\z&=&0\end{array}$$

2.2.34. Use the Gauss-Jordan method to solve the system of linear equations

$$\begin{array}{rcl}3x+2y-z&=&-16\\6x-4y+3z&=&12\\5x-2y+2z&=&4.\end{array}$$

$$\begin{bmatrix}3&2&-1&-16\\6&-4&3&12\\5&-2&2&4\end{bmatrix}\xrightarrow{\frac{1}{3}R_1\to R_1}\begin{bmatrix}1&\frac{2}{3}&-\frac{1}{3}&-\frac{16}{3}\\6&-4&3&12\\5&-2&2&4\end{bmatrix}$$

$$\xrightarrow{-6R_1+R_2\to R_2}\begin{bmatrix}1&\frac{2}{3}&-\frac{1}{3}&-\frac{16}{3}\\0&-8&5&44\\5&-2&2&4\end{bmatrix}\xrightarrow{-5R_1+R_3\to R_3}\begin{bmatrix}1&\frac{2}{3}&-\frac{1}{3}&-\frac{16}{3}\\0&-8&5&44\\0&-\frac{16}{3}&\frac{11}{3}&\frac{92}{3}\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{8}R_2\to R_2}\begin{bmatrix}1&\frac{2}{3}&-\frac{1}{3}&-\frac{16}{3}\\0&1&-\frac{5}{8}&-\frac{44}{8}\\0&-\frac{16}{3}&\frac{11}{3}&\frac{92}{3}\end{bmatrix}\xrightarrow{-\frac{2}{3}R_2+R_1\to R_1}\begin{bmatrix}1&0&\frac{2}{24}&-\frac{40}{24}\\0&1&-\frac{5}{8}&-\frac{44}{8}\\0&-\frac{16}{3}&\frac{11}{3}&\frac{92}{3}\end{bmatrix}$$

$$\xrightarrow{\frac{16}{3}R_2+R_3\to R_3}\begin{bmatrix}1&0&\frac{2}{24}&-\frac{40}{24}\\0&1&-\frac{5}{8}&-\frac{44}{8}\\0&0&-\frac{8}{24}&\frac{32}{24}\end{bmatrix}\xrightarrow{3R_3\to R_3}\begin{bmatrix}1&0&\frac{2}{24}&-\frac{40}{24}\\0&1&-\frac{5}{8}&-\frac{44}{8}\\0&0&1&4\end{bmatrix}$$

$$\xrightarrow{-\frac{2}{24}R_3+R_1\to R_1}\begin{bmatrix}1&0&0&-2\\0&1&-\frac{5}{8}&-\frac{44}{8}\\0&0&1&4\end{bmatrix}\xrightarrow{\frac{5}{8}R_3+R_2\to R_2}\begin{bmatrix}1&0&0&-2\\0&1&0&-3\\0&0&1&4\end{bmatrix}$$

$$\Leftrightarrow\begin{array}{rcl}x&=&-2\\y&=&-3\\z&=&4\end{array}$$

2.2.46. Pyro-Tech, Inc. is upgrading office technology by purchasing inkjet printers, LCD monitors, and additional memory chips. The total number of pieces of hardware purchased is 46. The cost of each inkjet printer is \$109, the cost of each LCD monitor is \$129, and the cost of each memory chip is \$89. The total amount of money spent on new hardware came to \$4774. They purchased two times as many memory chips as they did LCD monitors. Determine the number of each that was purchased.

Suppose that x is the number of printers, y is the number of monitors, and z is that of chips. From the number of pieces of hardware, we get $x + y + z = 46$. From the total cost, we obtain $109x + 129y + 89z = 4774$. Finally, from the last condition, we get another equation $2y = z$, or equivalently, $2y - z = 0$. Therefore we can make a matrix:

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 46 \\ 109 & 129 & 89 & 4774 \\ 0 & 2 & -1 & 0 \end{array}\right].$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 46 \\ 109 & 129 & 89 & 4774 \\ 0 & 2 & -1 & 0 \end{array}\right] \xrightarrow{-109R_1+R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 46 \\ 0 & 20 & -20 & -240 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{20}R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 46 \\ 0 & 1 & -1 & -12 \\ 0 & 2 & -1 & 0 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{cccc} 1 & 0 & 2 & 58 \\ 0 & 1 & -1 & -12 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3\to R_3} \left[\begin{array}{cccc} 1 & 0 & 2 & 58 \\ 0 & 1 & -1 & -12 \\ 0 & 0 & 1 & 24 \end{array}\right] \xrightarrow{-2R_3+R_1\to R_1} \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & -1 & -12 \\ 0 & 0 & 1 & 24 \end{array}\right]$$

$$\xrightarrow{R_3+R_2\to R_2} \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 24 \end{array}\right] \Leftrightarrow \begin{array}{rcl} x & = & 10 \\ y & = & 12 \\ z & = & 24 \end{array}$$

Therefore 10 printers, 12 monitors, and 24 memory chips were bought.

2.2.48. Nadir, Inc. produces three models of television sets: deluxe, super-deluxe, and ultra. Each deluxe se requires 2 hours of electronics work, 3 hours of assembly time, and 5 hours of finishing time. Each super-deluxe requires 1, 3, and 2 hours of electronics, assembly, and finishing time, respectively. Each ultra requires 2, 2, and 6 hours of the same work, respectively.

(a) There are 54 hours available for electronics, 72 hours available for assembly, and 148 hours available for finishing per week. How many of each model should be produced each week if all available time is to be used?

Suppose that x is the number of deluxe television sets, y is the number of super-deluxe television sets, and z is the number of ultra television sets. We have three equations:

$$\begin{array}{rcl} 2x + y + 2z & = & 54 \\ 3x + 3y + 2z & = & 72 \\ 5x + 2y + 6z & = & 148, \end{array}$$

which came from the number of hours for electronics, assembly, and finishing.

$$\left[\begin{array}{cccc} 2 & 1 & 2 & 54 \\ 3 & 3 & 2 & 72 \\ 5 & 2 & 6 & 148 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 3 & 3 & 2 & 72 \\ 5 & 2 & 6 & 148 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_2 \to R_2} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 5 & 2 & 6 & 148 \end{array}\right] \xrightarrow{-5R_1+R_3 \to R_3} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 0 & -\frac{1}{2} & 1 & 13 \end{array}\right]$$

$$\xrightarrow{\frac{2}{3}R_2 \to R_2} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{1}{2} & 1 & 13 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2+R_1 \to R_1} \left[\begin{array}{cccc} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{1}{2} & 1 & 13 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2+R_3 \to R_3} \left[\begin{array}{cccc} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & 0 & \frac{2}{3} & 10 \end{array}\right] \xrightarrow{\frac{3}{2}R_3 \to R_3} \left[\begin{array}{cccc} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & 0 & 1 & 15 \end{array}\right]$$

$$\xrightarrow{-\frac{4}{3}R_3+R_1 \to R_1} \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & 0 & 1 & 15 \end{array}\right] \xrightarrow{\frac{2}{3}R_3+R_2 \to R_2} \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 15 \end{array}\right]$$

$$\Leftrightarrow \begin{array}{rcl} x & = & 10 \\ y & = & 4 \\ z & = & 15 \end{array}$$

Therefore 10 deluxe, 4 super-deluxe, and 15 ultra models are produced.

(b) Suppose everything is the same as in part (a), but a super deluxe set requires 1, rather than 2, hours of finishing time. How many solutions are there now?
In this case, the third equation is changed to $5x + y + 6z = 148$. So we have a new matrix:

$$\left[\begin{array}{cccc} 2 & 1 & 2 & 54 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 148 \end{array}\right].$$

$$\left[\begin{array}{cccc} 2 & 1 & 2 & 54 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 148 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 148 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_2 \to R_2} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 5 & 1 & 6 & 148 \end{array}\right] \xrightarrow{-5R_1+R_3 \to R_3} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 0 & -\frac{3}{2} & 1 & 13 \end{array}\right]$$

$$\xrightarrow{\frac{2}{3}R_2 \to R_2} \left[\begin{array}{cccc} 1 & \frac{1}{2} & 1 & 27 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{3}{2} & 1 & 13 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2+R_1 \to R_1} \left[\begin{array}{cccc} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{3}{2} & 1 & 13 \end{array}\right]$$

$$\xrightarrow{\frac{3}{2}R_2+R_3\to R_3}\begin{bmatrix} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & 0 & 0 & 4 \end{bmatrix} \Leftrightarrow \begin{array}{rcl} x+\frac{4}{3}z & = & 30 \\ y-\frac{2}{3}z & = & -6 \\ 0 & = & 4 \end{array}$$

From the last equation, which is impossible, we can see that there is no solution.

(c) Suppose everything is the same as in part (b), but the total hours available for finishing changes from 148 hours to 144 hours. Now how many solutions are there?

The third equation becomes $5x+y+6z=144$. Therefore we have a matrix:

$$\begin{bmatrix} 2 & 1 & 2 & 54 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 144 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 1 & 2 & 54 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 144 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1\to R_1} \begin{bmatrix} 1 & \frac{1}{2} & 1 & 27 \\ 3 & 3 & 2 & 72 \\ 5 & 1 & 6 & 144 \end{bmatrix}$$

$$\xrightarrow{-3R_1+R_2\to R_2} \begin{bmatrix} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 5 & 1 & 6 & 144 \end{bmatrix} \xrightarrow{-5R_1+R_3\to R_3} \begin{bmatrix} 1 & \frac{1}{2} & 1 & 27 \\ 0 & \frac{3}{2} & -1 & -9 \\ 0 & -\frac{3}{2} & 1 & 9 \end{bmatrix}$$

$$\xrightarrow{\frac{2}{3}R_2\to R_2} \begin{bmatrix} 1 & \frac{1}{2} & 1 & 27 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{3}{2} & 1 & 9 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & -\frac{3}{2} & 1 & 9 \end{bmatrix}$$

$$\xrightarrow{\frac{3}{2}R_2+R_3\to R_3}\begin{bmatrix} 1 & 0 & \frac{4}{3} & 30 \\ 0 & 1 & -\frac{2}{3} & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Leftrightarrow \begin{array}{rcl} x+\frac{4}{3}z & = & 30 \\ y-\frac{2}{3}z & = & -6 \\ 0 & = & 0 \end{array}$$

So $y=\dfrac{2}{3}z-6$, $x=-\dfrac{4}{3}z+30$. Therefore a general solution is

$$(x,y,z)=(-\frac{4}{3}z+30, \frac{2}{3}z-6, z).$$

In this problem, a reasonable answer is nonnegative integer solution. So we have three inequalities

$$-\frac{4}{3}z+30\ge 0, \quad \frac{2}{3}z-6\ge 0, \quad z\ge 0.$$

$$-\frac{4}{3}z+30\ge 0 \Rightarrow 30\ge \frac{4}{3}z \Rightarrow z\le \frac{90}{4}=22.5$$

$$\frac{2}{3}z-6\ge 0 \Rightarrow \frac{2}{3}z\ge 6 \Rightarrow z\ge 9$$

Therefore $9 \le z \le 22.5$. Moreover, to get integral x and y, z must be a multiple of 3. Therefore the only possible values of z are $9, 12, 15, 18, 21$. So there are only 5 solutions.

2.2.54. A company produces three combinations of mixed vegetables that sell in 1-kg packages. Italian style combines 0.3 kg of zucchini, 0.3 of broccoli, and 0.4 of carrots. French style combines 0.6 kg of broccoli and 0.4 of carrots. Oriental style combines 0.2 kg of zucchini, 0.5 of broccoli, and 0.3 of carrots. The company has a stock of 16,200 kg of zucchini, 41,400 kg of broccoli, and 29,400 kg of carrots. How many packages of each style should it prepare to use up existing supplies?

2.3.22. Perform the indicated operations, where possible.

$$\begin{bmatrix} 1 & 5 \\ 2 & -3 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 8 & 5 \\ -1 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 \\ 2 & -3 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 8 & 5 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} 1+2 & 5+3 \\ 2+8 & -3+5 \\ 3+(-1) & 7+9 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 10 & 2 \\ 2 & 16 \end{bmatrix}$$

2.3.28. Perform the indicated operations, where possible.

$$\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4-1 & 3-1 \\ 1-1 & 2-0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3+1 & 2+1 \\ 0+1 & 2+4 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & 6 \end{bmatrix}$$

2.4.6. Let $A = \begin{bmatrix} -2 & 4 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 \\ 4 & 0 \end{bmatrix}$. Find $7B - 3A$.

$$7B - 3A = 7\begin{bmatrix} -6 & 2 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7\cdot(-6) & 7\cdot 2 \\ 7\cdot 4 & 7\cdot 0 \end{bmatrix} - \begin{bmatrix} 3\cdot(-2) & 3\cdot 4 \\ 3\cdot 0 & 3\cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} -42 & 14 \\ 28 & 0 \end{bmatrix} - \begin{bmatrix} -6 & 12 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} -42-(-6) & 14-12 \\ 28-0 & 0-9 \end{bmatrix} = \begin{bmatrix} -36 & 2 \\ 28 & -9 \end{bmatrix}$$

2.4.18. Find the matrix product, if possible.

$$\begin{bmatrix} 5 & 2 \\ 7 & 6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 5 & 2 \\ 7 & 6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 2 \end{bmatrix} &= \begin{bmatrix} 5 \cdot 1 + 2 \cdot 2 & 5 \cdot 4 + 2 \cdot (-1) & 5 \cdot 0 + 2 \cdot 2 \\ 7 \cdot 1 + 6 \cdot 2 & 7 \cdot 4 + 6 \cdot (-1) & 7 \cdot 0 + 6 \cdot 2 \\ 1 \cdot 1 + 0 \cdot 2 & 1 \cdot 4 + 0 \cdot (-1) & 1 \cdot 0 + 0 \cdot 2 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 18 & 4 \\ 19 & 22 & 12 \\ 1 & 4 & 0 \end{bmatrix} \end{aligned}$$

2.4.30. Find the matrix product, if possible.

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ -1 & 5 \end{bmatrix}$$

$$\begin{aligned} &\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ -1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2 \cdot 4 + (-2) \cdot 1 & 2 \cdot 3 + (-2) \cdot 2 \\ 1 \cdot 4 + (-1) \cdot 1 & 1 \cdot 3 + (-1) \cdot 2 \end{bmatrix} + \begin{bmatrix} 2 \cdot 7 + (-2) \cdot (-1) & 2 \cdot 0 + (-2) \cdot 5 \\ 1 \cdot 7 + (-1) \cdot (-1) & 1 \cdot 0 + (-1) \cdot 5 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 16 & -10 \\ 8 & -5 \end{bmatrix} = \begin{bmatrix} 22 & -8 \\ 11 & -4 \end{bmatrix} \end{aligned}$$