Homework 3 Solution

Section 2.5

2.5.14 Find the inverse, if it exists, for

$$\begin{bmatrix} -3 & -8 \\ 1 & 3 \end{bmatrix}.$$

$$\left[\begin{array}{cccc} -3 & -8 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc} 1 & 3 & 0 & 1 \\ -3 & -8 & 1 & 0 \end{array}\right] \xrightarrow{3R_1+R_2 \to R_2} \left[\begin{array}{cccc} 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{-3R_2+R_1 \to R_1} \left[\begin{array}{cccc} 1 & 0 & -3 & -8 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} -3 & -8 \\ 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -3 & -8 \\ 1 & 3 \end{bmatrix}.$$

2.5.18 Find the inverse, if it exists, for

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}.$$

$$\left[\begin{array}{cccccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_1+R_3 \to R_3} \left[\begin{array}{cccccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & -3 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{cccccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -3 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{-3R_2+R_1 \to R_1} \left[\begin{array}{cccccc} 1 & 0 & \frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & -3 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{3R_2+R_3 \to R_3} \left[\begin{array}{cccccc} 1 & 0 & \frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -1 & \frac{3}{2} & 1 \end{array}\right] \xrightarrow{2R_3 \to R_3} \left[\begin{array}{cccccc} 1 & 0 & \frac{3}{2} & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{-\frac{3}{2}R_3+R_1 \to R_1} \left[\begin{array}{cccccc} 1 & 0 & 0 & 4 & -6 & -3 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 & 3 & 2 \end{array}\right] \xrightarrow{\frac{1}{2}R_3+R_2 \to R_2} \left[\begin{array}{cccccc} 1 & 0 & 0 & 4 & -6 & -3 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & 3 & 2 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & -6 & -3 \\ -1 & 2 & 1 \\ -2 & 3 & 2 \end{bmatrix}$$

2.5.24 Find the inverse, if it exists, for

$$\begin{bmatrix} 4 & 1 & -4 \\ 2 & 1 & -1 \\ -2 & -4 & 5 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|ccc} 4 & 1 & -4 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ -2 & -4 & 5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{4}R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{4} & -1 & \frac{1}{4} & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ -2 & -4 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{4} & -1 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ -2 & -4 & 5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_1+R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{4} & -1 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{7}{2} & 3 & \frac{1}{2} & 0 & 1 \end{array}\right]$$

$$\xrightarrow{2R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{4} & -1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 2 & -1 & 2 & 0 \\ 0 & -\frac{7}{2} & 3 & \frac{1}{2} & 0 & 1 \end{array}\right] \xrightarrow{-\frac{1}{4}R_2+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & -1 & 2 & 0 \\ 0 & -\frac{7}{2} & 3 & \frac{1}{2} & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{7}{2}R_2+R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 10 & -3 & 7 & 1 \end{array}\right] \xrightarrow{\frac{1}{10}R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{3}{10} & \frac{7}{10} & \frac{1}{10} \end{array}\right]$$

$$\xrightarrow{\frac{3}{2}R_3+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{20} & \frac{11}{20} & \frac{3}{20} \\ 0 & 1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{3}{10} & \frac{7}{10} & \frac{1}{10} \end{array}\right] \xrightarrow{-2R_3+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{20} & \frac{11}{20} & \frac{3}{20} \\ 0 & 1 & 0 & -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{10} & \frac{7}{10} & \frac{1}{10} \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -4 \\ 2 & 1 & -1 \\ -2 & -4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{20} & \frac{11}{20} & \frac{3}{20} \\ -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ -\frac{3}{10} & \frac{7}{10} & \frac{1}{10} \end{bmatrix}$$

2.5.36 Solve

$$\begin{array}{rcrcrcl} 2x & + & y & & & = & 1 \\ & & 3y & + & z & = & 8 \\ 4x & - & y & - & 3z & = & 8 \end{array}$$

by using the inverse of the coefficient matrix if it exists and by the Gauss-Jordan method if the inverse doesn't exist.

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 4 & -1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 4 & -1 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-4R_1+R_3\to R_3}\begin{bmatrix}1&\frac{1}{2}&0&\frac{1}{2}&0&0\\0&3&1&0&1&0\\0&-3&-3&-2&0&1\end{bmatrix}\xrightarrow{\frac{1}{3}R_2\to R_2}\begin{bmatrix}1&\frac{1}{2}&0&\frac{1}{2}&0&0\\0&1&\frac{1}{3}&0&\frac{1}{3}&0\\0&-3&-3&-2&0&1\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_2+R_1\to R_1}\begin{bmatrix}1&0&-\frac{1}{6}&\frac{1}{2}&-\frac{1}{6}&0\\0&1&\frac{1}{3}&0&\frac{1}{3}&0\\0&-3&-3&-2&0&1\end{bmatrix}\xrightarrow{3R_2+R_3\to R_3}\begin{bmatrix}1&0&-\frac{1}{6}&\frac{1}{2}&-\frac{1}{6}&0\\0&1&\frac{1}{3}&0&\frac{1}{3}&0\\0&0&-2&-2&1&1\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_3\to R_3}\begin{bmatrix}1&0&-\frac{1}{6}&\frac{1}{2}&-\frac{1}{6}&0\\0&1&\frac{1}{3}&0&\frac{1}{3}&0\\0&0&1&1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}\xrightarrow{\frac{1}{6}R_3+R_1\to R_1}\begin{bmatrix}1&0&0&\frac{2}{3}&-\frac{1}{4}&-\frac{1}{12}\\0&1&\frac{1}{3}&0&\frac{1}{3}&0\\0&0&1&1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{3}R_3+R_2\to R_2}\begin{bmatrix}1&0&0&\frac{2}{3}&-\frac{1}{4}&-\frac{1}{12}\\0&1&0&-\frac{1}{3}&\frac{1}{2}&\frac{1}{6}\\0&0&1&1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}2&1&0\\0&3&1\\4&-1&-3\end{bmatrix}^{-1}=\begin{bmatrix}\frac{2}{3}&-\frac{1}{4}&-\frac{1}{12}\\-\frac{1}{3}&\frac{1}{2}&\frac{1}{6}\\1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2&1&0\\0&3&1\\4&-1&-3\end{bmatrix}^{-1}\begin{bmatrix}1\\8\\8\end{bmatrix}=\begin{bmatrix}\frac{2}{3}&-\frac{1}{4}&-\frac{1}{12}\\-\frac{1}{3}&\frac{1}{2}&\frac{1}{6}\\1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\8\\8\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{3}\cdot 1+(-\frac{1}{4})\cdot 8+(-\frac{1}{12})\cdot 8\\-\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 8+\frac{1}{6}\cdot 8\\1\cdot 1+(-\frac{1}{2})\cdot 8+(-\frac{1}{2})\cdot 8\end{bmatrix}=\begin{bmatrix}-2\\5\\-7\end{bmatrix}$$

$$x=-2,\ y=5,\ z=-7$$

2.5.60 An electronics company produces transistors, resistors, and computer chips. Each transistor requires 3 units of copper, 1 unit of zinc, and 2 units of glass. Each resistor requires 3, 2, and 1 units of the three materials, and each computer chip requires 2, 1, and 2 units of these materials, respectively. How many of each product can be made with the following amounts of materials?

(a) 810 units of copper, 410 units of zinc, and 490 units of glass

Suppose that x is the number of transistors, y is the number of resistors, and z is the number of chips.

From the amount of copper we use, we have $3x+3y+2z=810$. From that of zinc, we have $x+2y+z=410$. Finally, from the amount of glass, we obtain an equation $2x+y+2z=490$. Therefore we have a system of linear equations

$$\begin{aligned} 3x+3y+2z &= 810 \\ x+2y+z &= 410 \\ 2z+y+2z &= 490, \end{aligned}$$

or equivalently, a matrix equation

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 810 \\ 410 \\ 490 \end{bmatrix}.$$

Let's compute the inverse matrix.

$$\left[\begin{array}{cccccc} 3 & 3 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_2 \to R_2} \left[\begin{array}{cccccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -1 & 1 & -3 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2R_1+R_3 \to R_3} \left[\begin{array}{cccccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -1 & 1 & -3 & 0 \\ 0 & -3 & 0 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_2 \to R_2} \left[\begin{array}{cccccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 \\ 0 & -3 & 0 & 0 & -2 & 1 \end{array}\right] \xrightarrow{-2R_2+R_1 \to R_1} \left[\begin{array}{cccccc} 1 & 0 & \frac{1}{3} & \frac{2}{3} & -1 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 \\ 0 & -3 & 0 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{3R_2+R_3 \to R_3} \left[\begin{array}{cccccc} 1 & 0 & \frac{1}{3} & \frac{2}{3} & -1 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3+R_1 \to R_1} \left[\begin{array}{cccccc} 1 & 0 & 0 & 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_3+R_2 \to R_2} \left[\begin{array}{cccccc} 1 & 0 & 0 & 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 810 \\ 410 \\ 490 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 810 \\ 410 \\ 490 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 810 + (-\frac{4}{3}) \cdot 410 + (-\frac{1}{3}) \cdot 490 \\ 0 \cdot 810 + \frac{2}{3} \cdot 410 + (-\frac{1}{3}) \cdot 490 \\ -1 \cdot 810 + 1 \cdot 410 + 1 \cdot 490 \end{bmatrix} = \begin{bmatrix} 100 \\ 110 \\ 90 \end{bmatrix}$$

So we can produce 100 transistors, 110 resistors, and 90 chips.

(b) 765 units of copper, 385 units of zinc, and 470 units of glass

By the same procedure, we can make a matrix equation

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 765 \\ 385 \\ 470 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 765 \\ 385 \\ 470 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 765 + (-\frac{4}{3}) \cdot 385 + (-\frac{1}{3}) \cdot 470 \\ 0 \cdot 765 + \frac{2}{3} \cdot 385 + (-\frac{1}{3}) \cdot 470 \\ -1 \cdot 765 + 1 \cdot 385 + 1 \cdot 470 \end{bmatrix} = \begin{bmatrix} 95 \\ 100 \\ 90 \end{bmatrix}$$

Therefore we can produce 95 transistors, 100 resistors, and 90 chips.

(c) 1010 units of copper, 500 units of zinc, and 610 units of glass

By the same procedure, we can make a matrix equation

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1010 \\ 500 \\ 610 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1010 \\ 500 \\ 610 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 1010 + (-\frac{4}{3}) \cdot 500 + (-\frac{1}{3}) \cdot 610 \\ 0 \cdot 1010 + \frac{2}{3} \cdot 500 + (-\frac{1}{3}) \cdot 610 \\ -1 \cdot 1010 + 1 \cdot 500 + 1 \cdot 610 \end{bmatrix} = \begin{bmatrix} 140 \\ 130 \\ 100 \end{bmatrix}$$

Therefore we can produce 140 transistors, 130 resistors, and 100 chips.

2.5.62 Pretzels cost \$4 per lb, dried fruit \$5 per lb, and nuts \$9 per lb. The three ingredients are to be combined in a trail mix containing twice the weight of pretzels as dried fruit. How many pounds of each should be used to produce the following amounts at the given cost?

(a) 140 lb at \$6 per lb

Suppose that x is the weight of pretzels, y is the weight of dried fruit, and z is that of nuts. From the total weights, we have an equation $x + y + z = 140$. From the total cost, we have another equation $4x + 5y + 9z = 140 \cdot 6 = 840$.

Finally, because the weight of pretzels x has to be twice of y, we have $x = 2y$, or equivalently, $x - 2y = 0$. Therefore we have a system of linear equations

$$\begin{aligned} x + y + z &= 140 \\ 4x + 5y + 9z &= 840 \\ x - 2y &= 0. \end{aligned}$$

We can write it as a matrix equation

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 140 \\ 840 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 5 & 9 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-4R_1+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-R_1+R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 0 & -3 & -1 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{-R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & -4 & 5 & -1 & 0 \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 0 & -3 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{3R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & -4 & 5 & -1 & 0 \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 0 & 0 & 14 & -13 & 3 & 1 \end{bmatrix} \xrightarrow{\frac{1}{14}R_3 \to R_3} \begin{bmatrix} 1 & 0 & -4 & 5 & -1 & 0 \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 0 & 0 & 1 & -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix}$$

$$\xrightarrow{4R_3+R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ 0 & 1 & 5 & -4 & 1 & 0 \\ 0 & 0 & 1 & -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix} \xrightarrow{-5R_3+R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 & \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ 0 & 1 & 0 & \frac{9}{14} & -\frac{1}{14} & -\frac{5}{14} \\ 0 & 0 & 1 & -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 1 & -2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ \frac{9}{14} & -\frac{1}{14} & -\frac{5}{14} \\ -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 1 & -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 140 \\ 840 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ \frac{9}{14} & -\frac{1}{14} & -\frac{5}{14} \\ -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix} \begin{bmatrix} 140 \\ 840 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{7} \cdot 140 + (-\frac{1}{7}) \cdot 840 + \frac{2}{7} \cdot 0 \\ \frac{9}{14} \cdot 140 + (-\frac{1}{14}) \cdot 840 + (-\frac{5}{14}) \cdot 0 \\ -\frac{13}{14} \cdot 140 + \frac{3}{14} \cdot 840 + \frac{1}{14} \cdot 0 \end{bmatrix} = \begin{bmatrix} 60 \\ 30 \\ 50 \end{bmatrix}$$

Therefore 60 lb of pretzels, 30 lb of dried fruit, and 50 lb of nuts should be used.

(b) 100 lb at \$7.60 per lb

In this case, the total weight is $100 \cdot 7.60 = 760$. By the same idea, we can find a matrix equation

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 760 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ \frac{9}{14} & -\frac{1}{14} & -\frac{5}{14} \\ -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix} \begin{bmatrix} 140 \\ 840 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{7} \cdot 100 + (-\frac{1}{7}) \cdot 760 + \frac{2}{7} \cdot 0 \\ \frac{9}{14} \cdot 100 + (-\frac{1}{14}) \cdot 760 + (-\frac{5}{14}) \cdot 0 \\ -\frac{13}{14} \cdot 100 + \frac{3}{14} \cdot 760 + \frac{1}{14} \cdot 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 70 \end{bmatrix}$$

So 20 lb of pretzels, 10 lb of dried fruit, and 70 lb of nuts should be used.

(c) 125 lb at \$6.20 per lb

In this case, the total weight is $125 \cdot 6.20 = 775$. By the same idea, we can find a matrix equation

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ 775 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{9}{7} & -\frac{1}{7} & \frac{2}{7} \\ \frac{9}{14} & -\frac{1}{14} & -\frac{5}{14} \\ -\frac{13}{14} & \frac{3}{14} & \frac{1}{14} \end{bmatrix} \begin{bmatrix} 125 \\ 775 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{7} \cdot 125 + (-\frac{1}{7}) \cdot 775 + \frac{2}{7} \cdot 0 \\ \frac{9}{14} \cdot 125 + (-\frac{1}{14}) \cdot 775 + (-\frac{5}{14}) \cdot 0 \\ -\frac{13}{14} \cdot 125 + \frac{3}{14} \cdot 775 + \frac{1}{14} \cdot 0 \end{bmatrix} = \begin{bmatrix} 50 \\ 25 \\ 50 \end{bmatrix}$$

So 50 lb of pretzels, 25 lb of dried fruit, and 50 lb of nuts should be used.