Homework 4 Solution

Section 2.6

2.6.4. Find the production matrix for the following input-output and demand matrices using the open method.

$$A = \begin{bmatrix} 0.02 & 0.03 \\ 0.06 & 0.08 \end{bmatrix}, \quad D = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.02 & 0.03 \\ 0.06 & 0.08 \end{bmatrix} = \begin{bmatrix} 0.98 & -0.03 \\ -0.06 & 0.92 \end{bmatrix}$$

$$\begin{bmatrix} 0.98 & -0.03 & 1 & 0 \\ -0.06 & 0.92 & 0 & 1 \end{bmatrix} \overset{\frac{1}{0.98}R_1 \to R_1}{\longrightarrow} \begin{bmatrix} 1 & -\frac{3}{98} & \frac{1}{0.98} & 0 \\ -0.06 & 0.92 & 0 & 1 \end{bmatrix}$$

$$\overset{0.06R_1 + R_2 \to R_2}{\longrightarrow} \begin{bmatrix} 1 & -\frac{3}{98} & \frac{1}{0.98} & 0 \\ 0 & \frac{89.98}{98} & \frac{6}{98} & 1 \end{bmatrix} \overset{\frac{98}{89.98}R_2 \to R_2}{\longrightarrow} \begin{bmatrix} 1 & -\frac{3}{98} & \frac{1}{0.98} & 0 \\ 0 & 1 & \frac{6}{89.98} & \frac{98}{89.98} \end{bmatrix}$$

$$\overset{\frac{3}{98}R_2 + R_1 \to R_1}{\longrightarrow} \begin{bmatrix} 1 & 0 & \frac{9016}{98 \cdot 89.98} & \frac{3}{89.98} \\ 0 & 1 & \frac{6}{89.98} & \frac{98}{89.98} \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 1.0224 & 0.0333 \\ 0 & 1 & 0.0667 & 1.0891 \end{bmatrix}$$

100/98 + 18/(89.98*98) So

$$(I - A)^{-1} \approx \begin{bmatrix} 1.0224 & 0.0333 \\ 0.0667 & 1.0891 \end{bmatrix}.$$

Then the production matrix is

$$X = (I - A)^{-1}D \approx \begin{bmatrix} 1.0224 & 0.0333 \\ 0.0667 & 1.0891 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0224 \cdot 100 + 0.0333 \cdot 200 \\ 0.0667 \cdot 100 + 1.0891 \cdot 200 \end{bmatrix} = \begin{bmatrix} 108.9 \\ 224.49 \end{bmatrix}.$$

2.6.14. Suppose 1/3 unit of manufacturing (no agriculture or transportation) is required to produce 1 unit of agriculture, 1/4 unit of transportation is required to produce 1 unit of manufacturing, and 1/2 unit of agriculture is required to produce 1 unit of transportation. How many units of each commodity should be produced to satisfy a demand of 1000 units of each commodity?

We write three commodities in an order of agriculture, manufacturing, and transportation. Then the input-output matrix is

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{bmatrix}$$

and the demand matrix is

$$D = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix}.$$

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ -\frac{1}{3} & 1 & 0 \\ 0 & -\frac{1}{4} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{4} & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{3} & 1 & 0 \\ 0 & -\frac{1}{4} & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{4}R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{23}{24} & \frac{1}{12} & \frac{1}{4} & 1 \end{bmatrix} \xrightarrow{\frac{24}{23}R_3 \to R_3} \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{23} & \frac{6}{23} & \frac{24}{23} \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_3 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & \frac{24}{23} & \frac{3}{23} & \frac{12}{23} \\ 0 & 1 & -\frac{1}{6} & \frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{23} & \frac{6}{23} & \frac{24}{23} \end{bmatrix} \xrightarrow{\frac{1}{6}R_3 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 & \frac{24}{23} & \frac{3}{23} & \frac{12}{23} \\ 0 & 1 & 0 & \frac{8}{23} & \frac{24}{23} & \frac{4}{23} \\ 0 & 0 & 1 & \frac{2}{23} & \frac{6}{23} & \frac{24}{23} \end{bmatrix}$$

$$(I - A)^{-1} = \begin{bmatrix} \frac{24}{23} & \frac{3}{23} & \frac{12}{23} \\ \frac{8}{23} & \frac{24}{23} & \frac{4}{23} \\ \frac{2}{23} & \frac{6}{23} & \frac{24}{23} \end{bmatrix}$$

$$X = (I - A)^{-1}D = \begin{bmatrix} \frac{24}{23} & \frac{3}{23} & \frac{12}{23} \\ \frac{8}{23} & \frac{24}{23} & \frac{4}{23} \\ \frac{2}{23} & \frac{6}{23} & \frac{24}{23} \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} \frac{39000}{23} \\ \frac{36000}{23} \\ \frac{32000}{23} \end{bmatrix} \approx \begin{bmatrix} 1696 \\ 1565 \\ 1391 \end{bmatrix}$$

Therefore approximately 1696 units of agriculture, 1565 units of manufacturing, and 1391 units of transportation should be produced.