Homework 8 Solution

Section 7.3 $\sim$ 7.4

7.3.16. For the experiments that an unprepared student takes a three-question, true/false quiz in which he guesses the answers to all three questions, so each answer is equally likely to be correct or wrong, write out the sample space S, choosing an S with equally likely outcomes, if possible. Then give the value of $n(S)$ and tell whether the outcomes in S are equally likely. Finally, write the indicated events in set notation.

The sample space S is $\{ccc, ccw, cwc, cww, wcc, wcw, wwc, www\}$. So $n(S) = 8$. The outcomes are equally likely.

(a) The student gets three answers wrong.
$\{www\}$

(b) The student gets exactly two answers correct.
$\{ccw, cwc, wcc\}$

(c) The student gets only the first answer correct.
$\{cww\}$

7.3.22. A single fair die is rolled. Find the probabilities getting a number greater than 2.

$$\frac{4}{6} = \frac{2}{3}$$

7.3.36. A jar contains 3 white, 4 orange, 5 yellow, and 8 black marbles. If a marble is drawn at random, find the probability that it is orange.

$$\frac{4}{3+4+5+8} = \frac{4}{20} = \frac{1}{5}$$

7.3.54. The 2008 and the 2018 (projected) civilian labor forces by age given in the following table.

Age (in years)	2008 (in millions)	2018 (in millions)
16 to 24	22.0	21.1
25 to 54	104.4	105.9
55 and over	27.9	39.8
Total	154.3	166.8

(a) In 2008, find the probability that a member of the civilian labor force is age 55 or older.

$$\frac{27.9}{154.3} \approx 0.1808$$

(b) In 2018, find the probability that a member of the civilian labor force is age 55 or older.

$$\frac{39.8}{166.8} \approx 0.2386$$

(c) What do these projections imply about the future civilian labor force?

The portion of older labor forces is increasing.

7.3.56. For a medical experiment, people are classified as to whether they smoke, have a family history of heart disease, or are overweight. Define events E, F, and G as follows.

E : person smokes
F : person has a family history of heart disease
G : person is overweight

Describe each event in words.

(a) $E \cup F$

Person smokes or has a family history of heart disease, or both.

(b) $E' \cap F$

Person does not smoke and has a family history of heart disease.

(c) $F' \cup G'$

Person does not have a family history or heart disease or is not overweight, or both.

7.3.58. The projected U.S. population (in thousands) by race in 2020 and 2050 is given in the table.

Race	2020	2050
White	207,393	207,901
Hispanic	52,652	96,508
Black	41,538	53,555
Asian and Pacific Islander	18,557	32,432
Other	2602	3535

Find the probability that a randomly selected person in the given year is of the race specified.

(a) Hispanic in 2020

The total population in 2020 is $207,393+52,652+41,538+18,557+2,602 = 322,742$ (thousands). So the probability is $52,652/322,742 \approx 0.1631$.

(b) Hispanic in 2050

The total population in 2050 is $207,901+96,508+53,555+32,432+3,535 = 393,931$ (thousands). The probability is $96,508/393,931 \approx 0.2450$.

(c) Black in 2020

$$\frac{41538}{322742} \approx 0.1287$$

(d) Black in 2050

$$\frac{53555}{393931} \approx 0.1360$$

7.4.10. Two dice are rolled. Find the probabilities of rolling the given sums.

The total number of elements in the sample space is 36.

(a) 8

There are 5 ways to get the sum 8 ($\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$). So the probability is $5/36$.

(b) 9

There are 4 ways to get the sum 9 ($\{(3,6),(4,5),(5,4),(6,3)\}$). So the probability is $4/36 = 1/9$.

(c) 10

There are 3 ways to get the sum 10 ($\{(4,6),(5,5),(6,4)\}$). So the probability is $3/36 = 1/12$.

(d) 13

It is impossible to get the sum 13. Therefore the probability is zero.

7.4.16. One card is drawn from an ordinary deck of 52 cards. Find the probabilities of drawing the following cards.

(a) Less than a 4 (count aces as ones)

There are 4 aces, 4 twos and 4 threes.

$$\frac{4+4+4}{52} = \frac{12}{52} = \frac{3}{13}$$

(b) A diamond or a 7

Suppose that D is the event that the chosen card is a diamond, S is the event that the chosen card is a seven. Then $P(D) = 13/52 = 1/4$, $P(S) = 4/52 = 1/13$, and $P(D \cap S) = 1/52$.

$$P(D \cup S) = P(D) + P(S) - P(D \cap S) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

(c) A black card or an ace

Suppose that B is the event that the chosen card is a black, A is the event that the chosen card is an ace. Then $P(B) = 26/52 = 1/2$, $P(A) = 4/52 = 1/13$, and $P(B \cap A) = 2/52 = 1/26$.

$$P(B \cup A) = P(B) + P(A) - P(B \cap A) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

(d) A heart or a jack

Suppose that H is the event that the chosen card is a heart, J is the event that the chosen card is a jack. Then $P(H) = 13/52 = 1/4$, $P(J) = 4/52 = 1/13$, and $P(H \cap J) = 1/52$.

$$P(H \cup J) = P(H) + P(J) - P(H \cap J) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

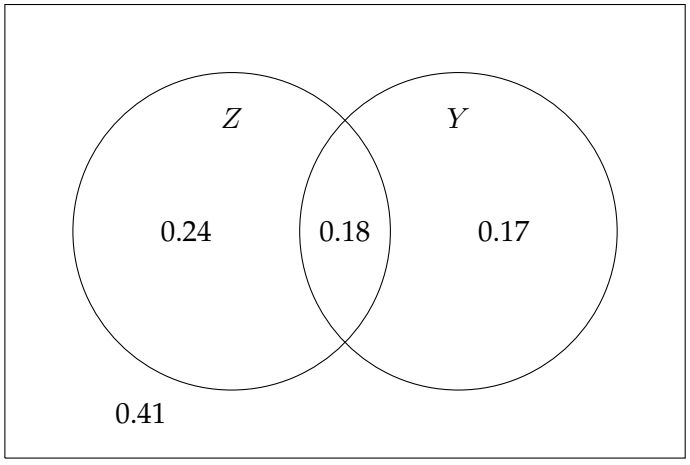
(e) A red card or a face card

Suppose that R is the event that the chosen card is a red, F is the event that the chosen card is a face card. Then $P(R) = 26/52 = 1/2$, $P(F) = 12/52 = 3/13$, and $P(R \cap F) = 6/52 = 3/26$.

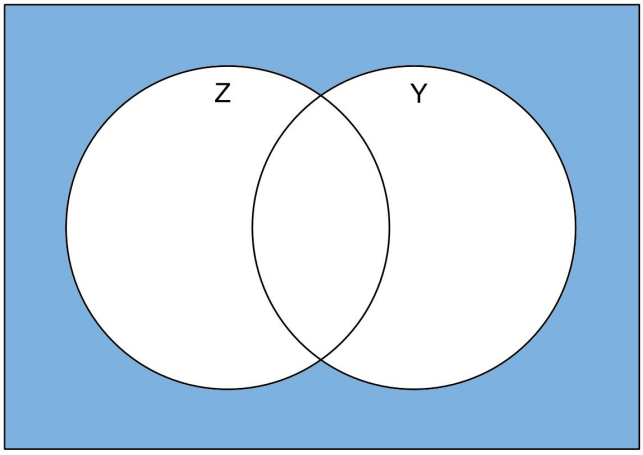
$$P(R \cup F) = P(R) + P(F) - P(R \cap F) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$$

7.4.22. Let $P(Z) = 0.42$, $P(Y) = 0.35$, and $P(Z \cup Y) = 0.59$. Find each probability.

$$P(Z \cup Y) = P(Z) + P(Y) - P(Z \cap Y) \Rightarrow 0.59 = 0.42 + 0.35 - P(Z \cap Y) \Rightarrow P(Z \cap Y) = 0.18$$

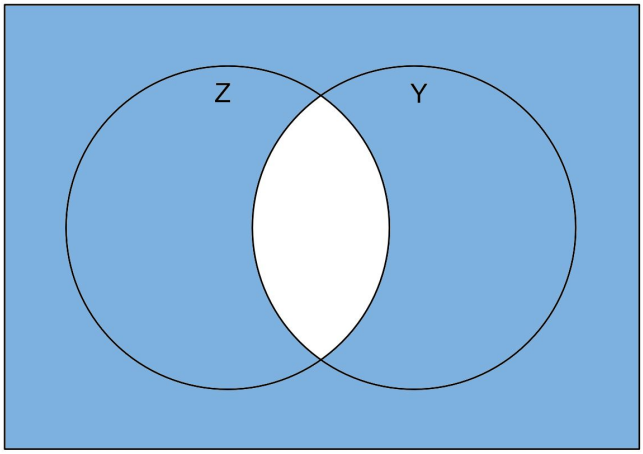


(a) $P(Z' \cap Y')$



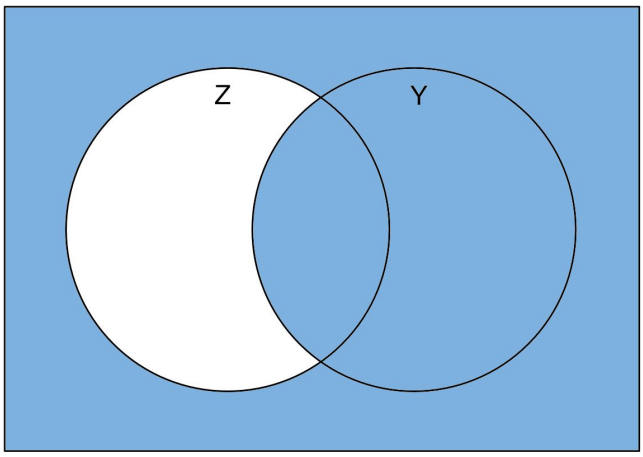
$$P(Z' \cap Y') = 0.41$$

(b) $P(Z' \cup Y')$



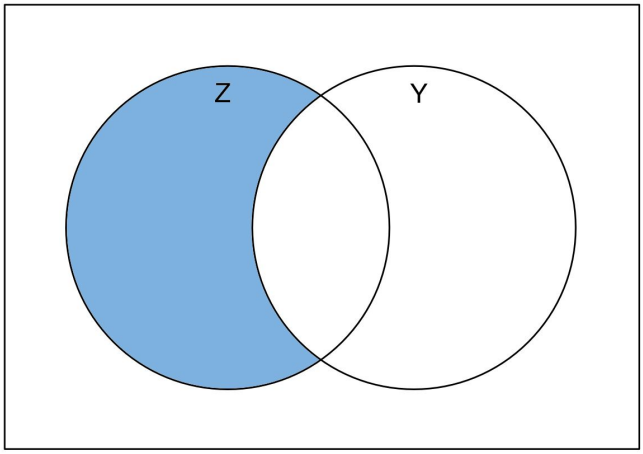
$$P(Z' \cup Y') = 0.41 + 0.24 + 0.17 = 0.82$$

(c) $P(Z' \cup Y)$



$$P(Z' \cup Y) = 0.41 + 0.17 + 0.16 = 0.76$$

(d) $P(Z \cap Y')$

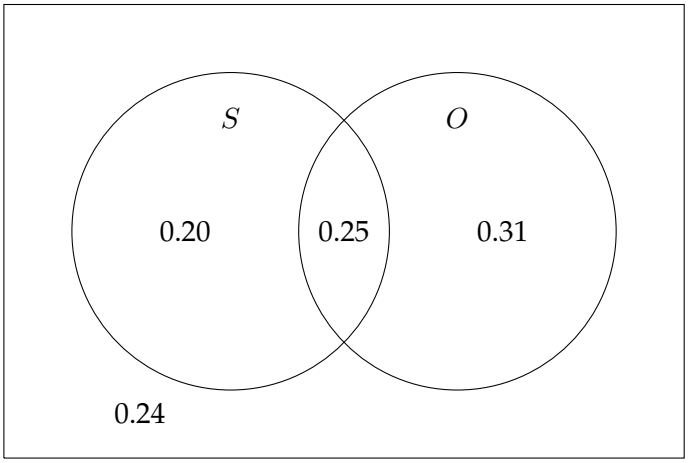


$$P(Z \cap Y') = 0.24$$

7.4.52. A study on body types gave the following results. 45% were short; 25% were short and overweight; and 24% were tall and not overweight. Find the probabilities that a person is the following.

(a) Overweight

Suppose that S is the event that the person is short, and O is the event that the person is overweight. Then $P(S) = 0.45$, $P(S \cap O) = 0.25$, $P(S' \cap O') = 0.24$.



$$P(O) = 0.25 + 0.31 = 0.56$$

(b) Short, but not overweight

$$P(S \cap O') = 0.20$$

(c) Tall and overweight

$$P(S' \cap O) = 0.31$$

7.4.62. A survey of 100 people about their music expenditures gave the following information: 38 bought rock music; 20 were teenagers who bought rock music; and 26 were teenagers. Find the probabilities that a person is:

(a) a teenager who buys non rock music;

Suppose that R is the event that the person buys rock music and T is the event that the person is a teenager. From the assumptions, we know that $P(R) = 38/100$, $P(T) = 26/100$, and $P(R \cap T) = 20/100$.

(b) someone who buys rock music or is a teenager;

$$P(R \cup T) = P(R) + P(T) - P(R \cap T) = \frac{38}{100} + \frac{26}{100} - \frac{20}{100} = \frac{44}{100} = 0.44$$

(c) not a teenager;

$$P(T') = 1 - P(T) = 1 - \frac{26}{100} = \frac{74}{100} = 0.74$$

(d) not a teenager, but a buyer of rock music.

$$P(T' \cap R) + P(T \cap R) = P(R)$$

$$\Rightarrow P(T' \cap R) + \frac{20}{100} = \frac{38}{100} \Rightarrow P(T' \cap R) = \frac{18}{100} = 0.18$$

7.4.64. At the first meeting of a committee to plan a local Lunar New Year celebration, the persons attending are 3 Chinese men, 4 Chinese women, 3 Vietnamese women, 2 Vietnamese men, 4 Korean women, and 2 Korean men. A chairperson is selected at random. Find the probabilities that the chairperson is the following.

(a) Chinese

There are $3+4+3+2+4+2=18$ people and among them, $3+4=7$ are Chinese. Thus the probability is $7/18$.

(b) Korean or a woman

Suppose that K is the event that the chairperson is Korean, and W is the event that the chairperson is a woman. $P(K)=6/18$, $P(W)=11/18$, $P(K\cap W)=4/18$.

$$P(K\cup W)=P(K)+P(W)-P(K\cap W)=\frac{6}{18}+\frac{11}{18}-\frac{4}{18}=\frac{13}{18}$$

(c) A man or Vietnamese

Suppose that M is the event that the chairperson is a man, and V is the event that the chairperson is a Vietnamese. $P(M)=7/18$, $P(V)=5/18$, $P(M\cap V)=2/18$.

$$P(M\cup V)=P(M)+P(V)-P(M\cap V)=\frac{7}{18}+\frac{5}{18}-\frac{2}{18}=\frac{10}{18}=\frac{5}{9}$$

(d) Chinese or Vietnamese

Suppose that C is the event that the chairperson is a Chinese.

$$P(C\cup V)=P(C)+P(V)-P(C\cap V)=\frac{7}{18}+\frac{5}{18}-0=\frac{12}{18}=\frac{2}{3}$$

(e) Korean and a woman

$$P(K\cap W)=\frac{4}{18}=\frac{2}{9}$$