Homework 9 Solution

Section 7.5 $\sim$ 7.6.

7.5.6. If two fair dice are rolled, find the probability that the sum is 6, given that the roll was a "double" (two identical numbers).

There are 6 ways to get a double ($\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$). Among them, $(3,3)$ has the sum 6. Therefore the probability is $1/6$.

7.5.12. If two cards are drawn without replacement from an ordinary deck, find the probability that the second is an ace, given that the first is not an ace.

If the first card is not an ace, then 4 of the remaining 51 cards are aces. Therefore the probability is $4/51$.

7.5.30. If A and B are events such that $P(A) = 0.5$ and $P(A \cup B) = 0.7$, find $P(B)$ when

(a) A and B are mutually exclusive;
If A and B are mutually exclusive, $P(A \cap B) = 0$. Then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$$

$$\Rightarrow 0.7 = 0.5 + P(B) \Rightarrow P(B) = 0.2$$

(b) A and B are independent.
If A and B are independent, $P(A \cap B) = P(A)P(B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow 0.7 = 0.5 + P(B) - 0.5P(B) = 0.5 + 0.5P(B)$$

$$\Rightarrow 0.5P(B) = 0.2 \Rightarrow P(B) = 0.4$$

7.5.74. In a letter to the journal *Nature*, Robert A. J. Matthews gives the following table of outcomes of forecast and weather over 1000 1-hour walks, based on the United Kingdom's Meteorological office's 83% accuracy in 24-hour forecasts.

	Rain	No Rain	Totals
Forecast of Rain	66	156	222
Forecast of No Rain	14	764	778
Totals	80	920	1000

(a) Verify that the probability that the forecast called for rain, given that there was rain, is indeed 83%. Also verify that the probability that the forecast called for no rain, given that there was no rain, is also 83%.

The probability that the forecast called for rain, given that there was rain is $\frac{66}{80} = 0.825 \approx 0.83$. The probability that the forecast called for no rain, given that there was no rain, is $\frac{764}{920} \approx 0.83$.

(b) Calculate the probability that there was rain, given that the forecast called for rain.

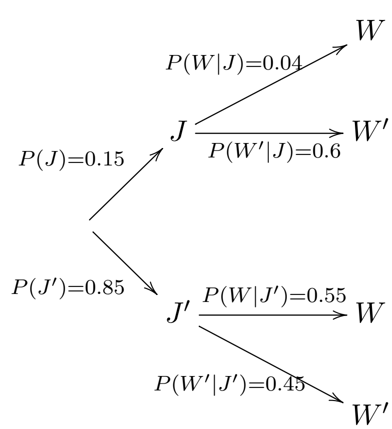
$$\frac{66}{222} \approx 0.2973$$

(c) Calculate the probability that there was no rain, given that the forecast called for no rain.

$$\frac{764}{778} \approx 0.9820$$

7.5.80. In a certain area, 15% of the population are joggers and 40% of the joggers are women. If 55% of those who do not jog are women, find the probabilities that an individual from that community fits the following descriptions.

Suppose that J is the event that a randomly chosen person is a jogger, and W is the event that the person is a woman. Then $P(J) = 0.15$, $P(W|J) = 0.4$, and $P(W|J') = 0.55$.



(a) A woman jogger

$$P(J \cap W) = P(J)P(W|J) = 0.15 \cdot 0.4 = 0.06$$

(b) A man who is not a jogger

$$P(W' \cap J') = P(J')P(W|J') = (1 - P(J))0.55 = (1 - 0.15)0.55 = 0.4675$$

(c) A woman

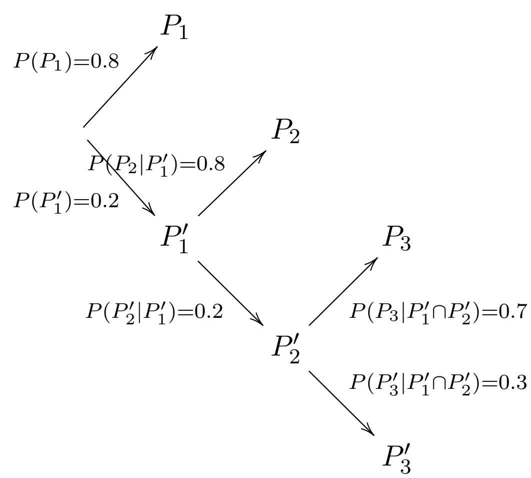
$$P(W) = P(W \cap J) + P(W \cap J') = P(W \cap J) + P(J')P(W|J')$$
$$= 0.06 + 0.85 \cdot 0.55 = 0.5275$$

(d) Are the events that a person is a woman and a person is a jogger independent? Explain.

$P(J \cap W) = 0.06$, but $P(J)P(W) = 0.15 \cdot 0.5275 \approx 0.0791$. Therefore these two events are not independent.

7.5.82. The Motor Vehicle Department has found that the probability of a person passing the test for a driver's license on the first try is 0.75. The probability that an individual who fails on the first test will pass on the second try is 0.80, and the probability that an individual who fails the first and second tests will pass the third time is 0.70. Find the probabilities that an individual will do the following.

Suppose that P_1 is the event that a person passes the first test, P_2 is the event that a person passes the second test, etc. Then $P(P_1) = 0.75$, $P(P_2|P_1') = 0.8$, $P(P_3|P_1' \cap P_2') = 0.7$.



(a) Fail both the first and second test

$$P(P_1' \cap P_2') = P(P_1')P(P_2'|P_1') = 0.25 \cdot 0.2 = 0.05$$

(b) Fail three times in a row

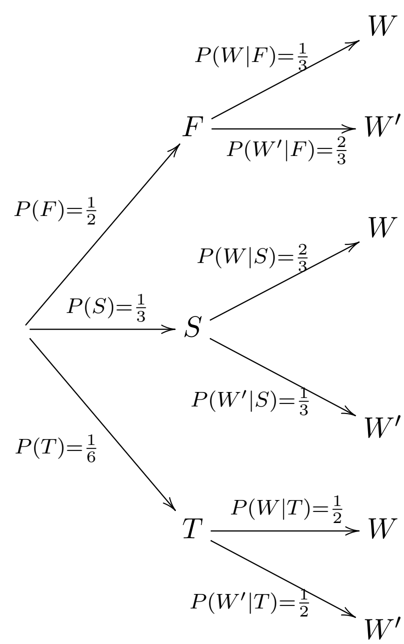
$$P(P_1' \cap P_2' \cap P_3') = P(P_1' \cap P_2')P(P_3'|P_1' \cap P_2') = 0.05 \cdot 0.3 = 0.0015$$

(c) Require at least two tries

$$P(P_1') = 1 - P(P_1) = 1 - 0.75 = 0.25$$

7.6.8. Suppose you have three jars with the following contents: 2 black balls and 1 white ball in the first, 1 black ball and 2 white balls in the second, and 1 black ball and 1 white ball in the third. One jar is to be selected, and then 1 ball is to be drawn from the selected jar If the probabilities of selecting the first, second, or third jar are $1/2$, $1/3$, and $1/6$, respectively, find the probabilities that if a white ball is drawn, it came from the third jar.

Suppose that F (S, T respectively) is the event that you chose the first (second, and third respectively) jar. Then $P(F) = 1/2$, $P(S) = 1/3$, and $P(T) = 1/6$. Also if we denote by W the probability that you picked a white ball, then $P(W|F) = 1/3$, $P(W|S) = 2/3$ and $P(W|T) = 1/2$.



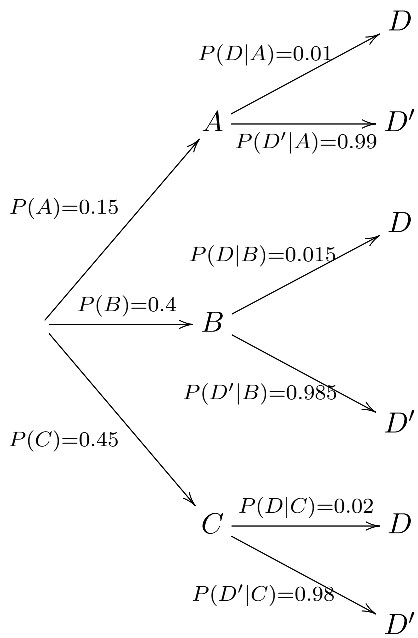
$$P(T \cap W) = P(T)P(W|T) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

$$\begin{aligned} P(W) &= P(F \cap W) + P(S \cap W) + P(T \cap W) \\ &= P(F)P(W|F) + P(S)P(W|S) + P(T)P(W|T) \\ &= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{2} = \frac{17}{36} \end{aligned}$$

$$P(T|W) = \frac{P(T \cap W)}{P(W)} = \frac{\frac{1}{12}}{\frac{17}{36}} = \frac{3}{17}$$

7.6.14. Companies A, B, and C produce 15%, 40%, and 45%, respectively, of the major appliances sold in a certain area. In that area, 1% of the company A appliances, 1.5% of the company B appliances, and 2% of the company C appliances need service within the first year. Suppose a defective appliance is chosen at random; find the probabilities that it was manufactured by company B.

Suppose that D is the event that the chosen item is a defective one.



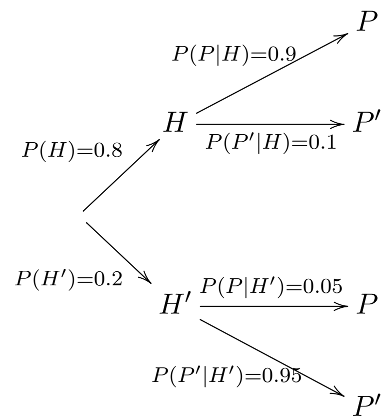
The probability we would like to find is $P(B|D)$.

$$P(B \cap D) = P(B)P(D|B) = 0.4 \cdot 0.015 = 0.006$$

$$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$$
$$= P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)$$
$$= 0.15 \cdot 0.01 + 0.4 \cdot 0.015 + 0.45 \cdot 0.02 = 0.0165$$
$$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.006}{0.0165} \approx 0.3636$$

7.6.24. The probability that a person with certain symptoms has hepatitis is 0.8. The blood test used to confirm this diagnosis gives positive results for 90% of people with the disease and 5% of those without the disease. What is the probability that an individual who has the symptoms and who reacts positively to the test actually has hepatitis?

Suppose that H is the event that the person has hepatitis, and P is the event that the test was positive.



We want to find $P(H|P)$.

$$P(H \cap P) = P(H)P(P|H) = 0.8 \cdot 0.9 = 0.72$$

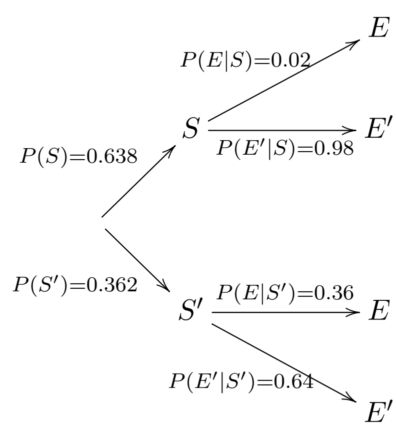
$$P(P) = P(H \cap P) + P(H' \cap P) = P(H)P(P|H) + P(H')P(P|H')$$

$$= 0.8 \cdot 0.9 + 0.2 \cdot 0.05 = 0.73$$

$$P(H|P) = \frac{P(H \cap P)}{P(P)} = \frac{0.72}{0.73} \approx 0.9863$$

7.6.36. A 2009 federal study showed that 63.8% of occupants involved in a fatal car crash wore seat belts. Of those in a fatal crash who wore seat belts, 2% were ejected from the vehicle. For those not wearing seat belts, 36% were ejected from the vehicle.

Suppose that S is the event that an occupant involved in a fatal car crash wore a seat belt, and E is the event that the occupant was ejected from the vehicle.



(a) Find the probability that a randomly selected person in a fatal car crash who was ejected from the vehicle was wearing a seat belt.

We want to find $P(S|E)$.

$$P(S \cap E) = P(S)P(E|S) = 0.638 \cdot 0.02 \approx 0.01276$$

$$P(E) = P(S \cap E) + P(S' \cap E) = P(S)P(E|S) + P(S')P(E|S')$$
$$= 0.638 \cdot 0.02 + 0.362 \cdot 0.36 = 0.14308$$
$$P(S|E) = \frac{P(S \cap E)}{P(E)} \approx 0.0892$$

(b) Find the probability that a randomly selected person in a fatal car crash who was not ejected from the vehicle was not wearing a seat belt.

The probability we want to find is $P(S'|E')$.

$$P(S' \cap E') = P(S')P(E'|S') = 0.362 \cdot 0.64 = 0.23168$$

$$P(E') = P(S \cap E') + P(S' \cap E') = P(S)P(E'|S) + P(S')P(E'|S')$$
$$= 0.638 \cdot 0.98 + 0.362 \cdot 0.64 = 0.85692$$
$$P(S'|E') = \frac{P(S' \cap E')}{P(E')} \approx 0.2704$$