MATH 1100 MIDTERM EXAM 1 SOLUTION

SPRING 2015 - MOON

Name: ______________________________

- Write your answer neatly and show steps.
- Except calculators, any electronic devices including laptops and cell phones are not allowed.
- Do not use the graphing function on your calculator.

(1) Quick survey.

(a) (1pt) This class is:

Too easy			Moderate			Too difficult
1	2	3	4	5	6	7

(b) (2pts) Write any suggestion for improving this class. (For instance, give more examples in class, explain proofs of formulas in detail, give more homework, slow down the tempo, ...)

(2) (4 pts) Let

$$A = \begin{bmatrix} 3 & 2 & 1 & -3 \\ 2 & 0 & 5 & 9 \\ 6 & -2 & 0 & 10 \end{bmatrix}.$$

Write the result of the transformation $-2R_1 + R_3 \to R_3$ for A.

$$\begin{bmatrix} 3 & 2 & 1 & -3 \\ 2 & 0 & 5 & 9 \\ -2\cdot 3+6 & -2\cdot 2+(-2) & (-2)\cdot 1+0 & -2\cdot(-3)+10 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 & -3 \\ 2 & 0 & 5 & 9 \\ 0 & -6 & -2 & 16 \end{bmatrix}$$

- Making computational mistake: -1 pt for each entry.
- Writing third row only: -2 pts.

Date: February 23, 2014.

(3) Suppose that

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 1 & -4 \end{bmatrix}.$$

(a) (4 pts) Compute $3A + B$.

$$3A + B = 3\begin{bmatrix} 2 & 5 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 6 & 15 \\ -9 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ -8 & -4 \end{bmatrix}$$

- Computing $3A$: 2 pts.
- Finding the answer $\begin{bmatrix} 9 & 15 \\ -8 & -4 \end{bmatrix}$: 4 pts.

(b) (4 pts) Compute AB.

$$AB = \begin{bmatrix} 2 & 5 \\ -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 3 + 5 \cdot 1 & 2 \cdot 0 + 5 \cdot (-4) \\ -3 \cdot 3 + 0 \cdot 1 & -3 \cdot 0 + 0 \cdot (-4) \end{bmatrix} = \begin{bmatrix} 11 & -20 \\ -9 & 0 \end{bmatrix}$$

- Making a computational mistake: -1 pt for each entry.

(4) A dietitian in a hospital is to arrange a special diet composed of three basic foods. The diet is to include exactly 18 units of iron, 34 units of calcium, and 22 units of vitamin A. The number of units per ounce of each special ingredient for each of the foods is indicated in the table.

	units per ounce		
	food A	food B	food C
iron	1	1	2
calcium	3	1	2
vitamin A	1	3	2

(a) (3 pts) Set up a system of linear equations and corresponding augmented matrix for this problem. Write the meaning of each variable.

x: the number of ounces of food A
y: the number of ounces of food B
z: the number of ounces of food C
System of linear equations:

$$\begin{aligned} x + y + 2z &= 18 \\ 3x + y + 2z &= 34 \\ x + 3y + 2z &= 22 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 2 & 18 \\ 3 & 1 & 2 & 34 \\ 1 & 3 & 2 & 22 \end{bmatrix}$$

- Writing system of linear equations: 2 pts.
- Indicating the meaning of each variable correctly: 1 pt.

(b) (8 pts) *By using Gauss-Jordan method*, solve the system of linear equations in (a). Write all steps and all transformations you use.

$$\begin{bmatrix} 1 & 1 & 2 & 18 \\ 3 & 1 & 2 & 34 \\ 1 & 3 & 2 & 22 \end{bmatrix} \xrightarrow{-3R_1+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 2 & 18 \\ 0 & -2 & -4 & -20 \\ 1 & 3 & 2 & 22 \end{bmatrix}$$

$$\xrightarrow{-R_1+R_3 \to R_3} \begin{bmatrix} 1 & 1 & 2 & 18 \\ 0 & -2 & -4 & -20 \\ 0 & 2 & 0 & 4 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 1 & 2 & 18 \\ 0 & 1 & 2 & 10 \\ 0 & 2 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 2 & 10 \\ 0 & 2 & 0 & 4 \end{bmatrix} \xrightarrow{-2R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & -4 & -16 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{4}R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 1 & 4 \end{bmatrix} \xrightarrow{-2R_3+R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\Rightarrow x = 8, y = 2, z = 4$$

- Finding the correct answer $x = 8, y = 2, z = 4$ with appropriate transformations: 8 pts.
- Performing an incorrect transformation: -2 pts each.
- Without using Gauss-Jordan method, you can get at most 6 pts.

(c) (2 pts) How many ounces of each food must be used to meet the diet requirements?

Food A: 8 ounces, Food B: 2 ounces, Food C: 4 ounces

(5) On an amusement park, parking fees are \$5 for local residents and \$7 for all others. At the end of each day, the total number of vehicles parked that day and the gross receipts for the day are recorded, but the number of vehicles in each category is not. The following table contains the information for the last weekend.

	Friday	Saturday	Sunday
Vehicle parked	1,200	1,550	1,740
Gross receipts	\$7,700	\$9,794	\$10,094

(a) (2 pts) Write a system of linear equations for the number of vehicles of local residents and that of the others on the Friday.

x: number of vehicles of local residents
y: number of vehicles of others

$$\begin{aligned} x+y &= 1200 \\ 5x+7y &= 7700 \end{aligned}$$

(b) (1 pts) Write the system of linear equations in (a) as a matrix equation.

$$\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1200 \\ 7700 \end{bmatrix}$$

(c) (7 pts) *By using the inverse matrix method*, Find the solution of (a). You can't get any credit if you don't use the inverse matrix method.

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{array}\right] \xrightarrow{-5R_1+R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 2 & -5 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{5}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{cccc} 1 & 0 & \frac{7}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{5}{2} & \frac{1}{2} \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{7}{2} & -\frac{1}{2} \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 1200 \\ 7700 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & -\frac{1}{2} \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1200 \\ 7700 \end{bmatrix} \\ &= \begin{bmatrix} \frac{7}{2}\cdot 1200 + \left(-\frac{1}{2}\right)\cdot 7700 \\ -\frac{5}{2}\cdot 1200 + \frac{1}{2}\cdot 7700 \end{bmatrix} = \begin{bmatrix} 350 \\ 850 \end{bmatrix} \end{aligned}$$

$$\Rightarrow x = 350, y = 850$$

The number of vehicles of local residents: 350
The number of vehicles of the others: 850

- Finding the inverse matrix $\begin{bmatrix} \frac{7}{2} & -\frac{1}{2} \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix}$: 5 pts.
- Getting the correct answer 350 and 850: 7 pts.

(d) (5 pts) How many vehicles in each category used the park's parking facilities, on Saturday and Sunday?

Saturday:

$$\begin{bmatrix} \frac{7}{2} & -\frac{1}{2} \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1550 \\ 9794 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \cdot 1550 + \left(-\frac{1}{2}\right) \cdot 9794 \\ -\frac{5}{2} \cdot 1550 + \frac{1}{2} \cdot 9794 \end{bmatrix} = \begin{bmatrix} 528 \\ 1022 \end{bmatrix}$$

528 vehicles of local residents, and 1022 vehicles of the others.

Sunday:

$$\begin{bmatrix} \frac{7}{2} & -\frac{1}{2} \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1740 \\ 10094 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \cdot 1740 + \left(-\frac{1}{2}\right) \cdot 10094 \\ -\frac{5}{2} \cdot 1740 + \frac{1}{2} \cdot 10094 \end{bmatrix} = \begin{bmatrix} 1043 \\ 697 \end{bmatrix}$$

1043 vehicles of local residents, and 697 vehicles of the others.

- Although the inverse matrix computed in the last problem was wrong, if you know how to use it to find the answer, you can earn 4 pts.
- Otherwise, you can get the credit only if the answer was correct.

(6) An economy is based on three commodities, agriculture, energy, and manufacturing. Production of a unit of agriculture requires an input of 0.2 units of agriculture and 0.4 units of energy. Production of a unit of energy requires an input of 0.2 units of energy and 0.4 units of manufacturing. Production of a unit of manufacturing requires 0.1 units of agriculture, 0.1 units of energy, and 0.3 units of manufacturing. We want to find the production level satisfying a demand of 20 units of agriculture, 10 units of energy, and 30 units of manufacturing.

(a) (3 pts) Find the input-output matrix A. (In the order of agriculture, energy, and manufacturing,)

$$\begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0 & 0.4 & 0.3 \end{bmatrix}$$

(b) (1 pt) Find the demand matrix D.

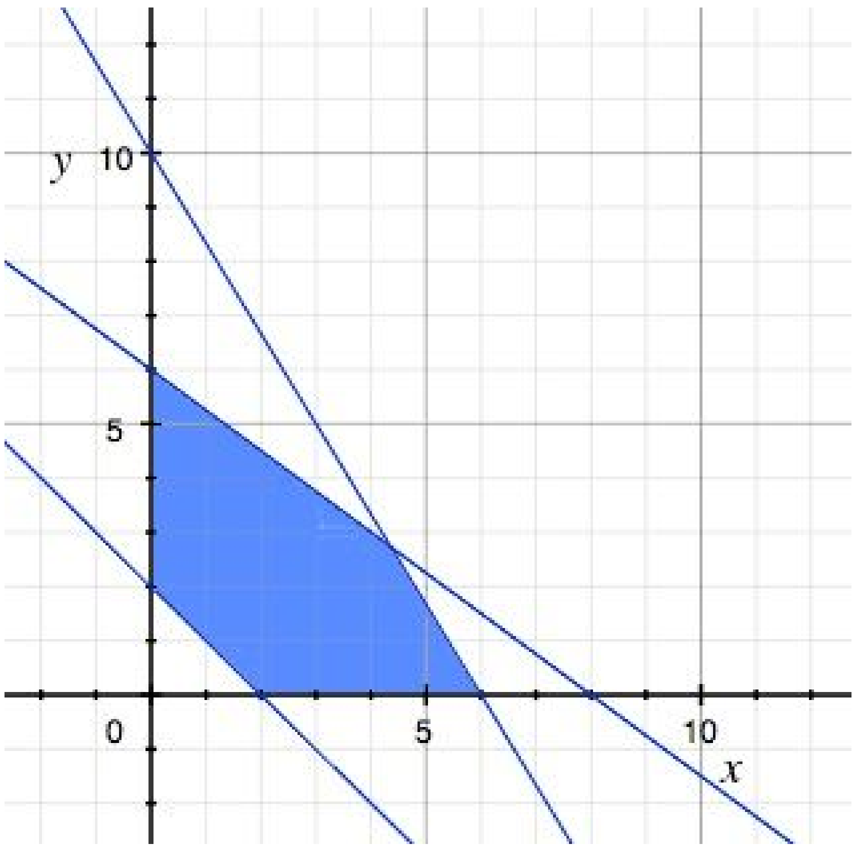
$$\begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix}$$

(c) (3 pts) Explain how to find the production level. You don't need to find the answer. Just describe what you need to compute, and how to evaluate it.

The production level is given by a matrix X satisfying $X - AX = D$, or equivalently, $(I - A)X = D$. So we should compute $I - A$ and its inverse $(I - A)^{-1}$. After that, $X = (I - A)^{-1}D$.

(7) (10 pts) On the plane below, sketch the feasible region of the following system of linear inequalities:

$$5x + 3y \leq 30$$
$$3x + 4y \leq 24$$
$$x + y \geq 2$$
$$x \geq 0$$
$$y \geq 0$$



- Sketching equations of lines $5x + 3y = 30$, $3x + 4y = 24$, $x + y = 2$: 2 pts each.
- Taking the half planes given by inequalities $5x + 3y \leq 30$, $3x + 4y \leq 24$, and $x + y \geq 2$: 1 pt each.
- Getting the correct feasible region: 10 pts.
- If it is unclear that where is the feasible region in your answer, -3 pts.