MATH 1100 MIDTERM EXAM 2 SOLUTION

SPRING 2015 - MOON

(1) Suppose that the universal set U is $\{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,3,5,7,9\}$, and $B = \{2,3,4,5,6,7,8\}$.

(a) (2 pts) Find $A \cup B$.

$$A \cup B = \{1,2,3,4,5,6,7,8,9\}$$

(b) (2 pts) Find A'.

$$A' = \{2,4,6,8,10\}$$

(c) (2 pts) Find $n(B)$.

$$n(B) = 7$$

- $\{7\}$: -1 pt.

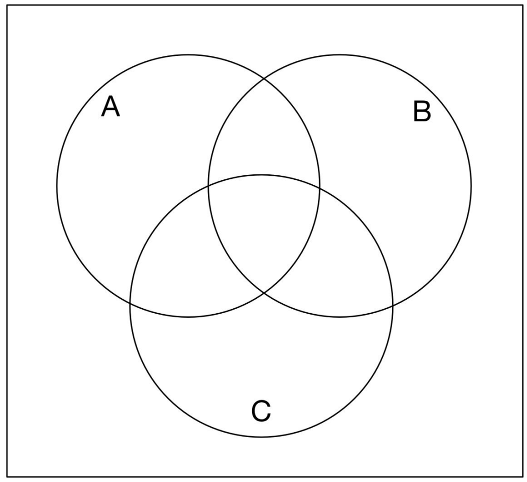
(d) (2 pts) Find $B \cap B'$.

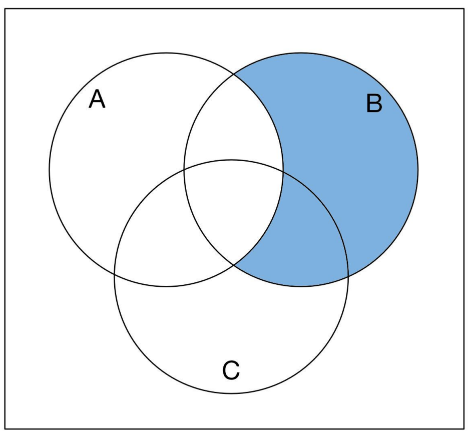
$$B \cap B' = \emptyset$$

- $\{\}$ or $\{\emptyset\}$: -1 pt.

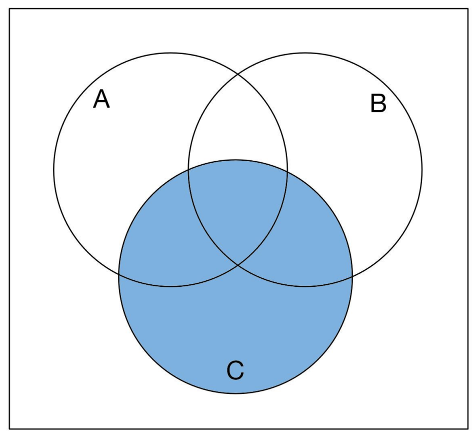
Date: April 13, 2014.

(2) (5 pts) Use shading to show the set $(A' \cap B) \cup C$ on the Venn diagram below.

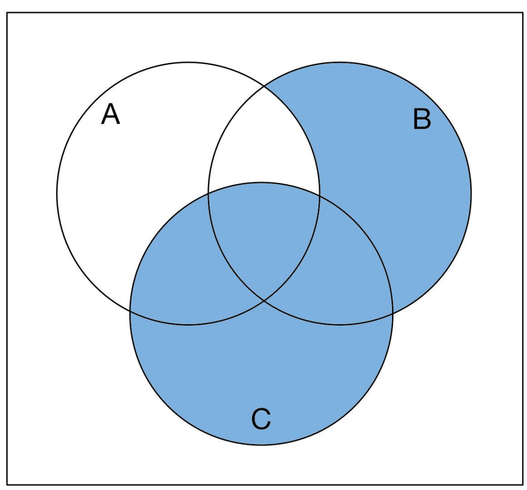




$A' \cap B$



C



$(A' \cap B) \cap C$

- Sketching the Venn diagram of $A' \cap B$ correctly: 3 pts.

(3) A bakery makes cakes and cookies. Each batch of cakes requires 1 hour in the oven and 3 hours in the decorating room. Each batch of cookies needs 3 hours in the oven and 3 hours in the decorating room. The oven is available no more than 12 hours per day, and the decorating room can be used no more than 18 hours per day. The bakery sells a batch of cakes as \$400, and a batch of cookies as \$250. Sheldon, the owner of the bakery, wants to find the possible maximum revenue.

(a) (2 pts) Identify the variables and write the constraints as inequalities.

x: the number of batches of cakes
y: the number of batches of cookies

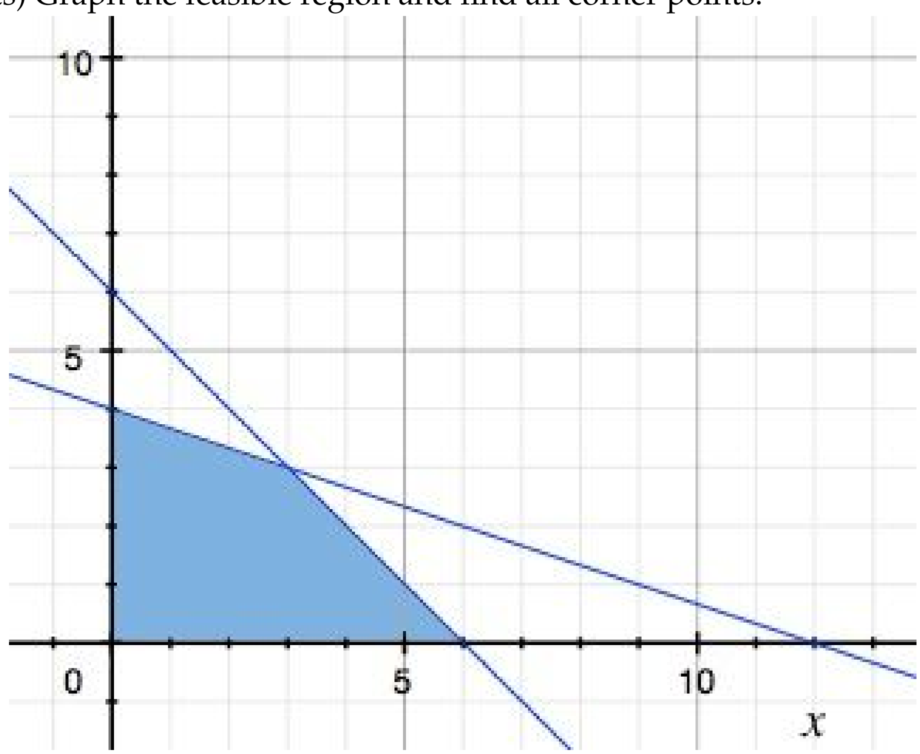
$$\begin{aligned} x + 3y &\leq 12 \\ 3x + 3y &\leq 18 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

- Introducing appropriate variables: 1 pt.
- Finding all inequalities describing the feasible region: 1 pt.

(b) (2 pts) Write the objective function.

$$z = 400x + 250y$$

(c) (5 pts) Graph the feasible region and find all corner points.



$$\begin{array}{rcl} x+3y &=& 12 \\ 3x+3y &=& 18 \end{array} \Rightarrow \begin{array}{rcl} x+3y &=& 12 \\ x+y &=& 6 \end{array}$$

$$\Rightarrow \begin{array}{rcl} 2y &=& 6 \\ x+y &=& 6 \end{array} \Rightarrow y=3, x=3$$

Corner points: $(0,0), (6,0), (0,4), (3,3)$

- Sketching the feasible region: 3 pts.
- Finding all corner points: 2 pts.

(d) (2 pts) Find the maximum revenue and determine how many batches of cakes and cookies he needs to prepare.

corner point	objective function $(400x + 250y)$
$(0,0)$	$400 \cdot 0 + 250 \cdot 0 = 0$
$(6,0)$	$400 \cdot 6 + 250 \cdot 0 = 2400$
$(0,4)$	$400 \cdot 0 + 250 \cdot 4 = 1000$
$(3,3)$	$400 \cdot 3 + 250 \cdot 3 = 1950$

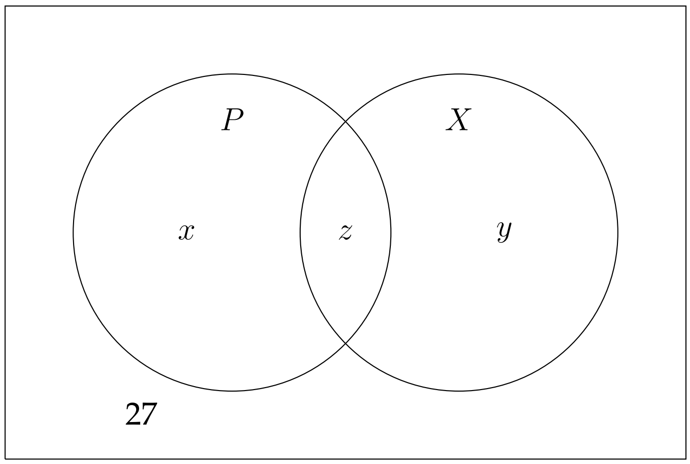
Therefore the maximum revenue is $2,400 and it occurs when the bakery prepares 6 batches of cakes only.

- Finding the maximum revenue $2,400: 1 pt.
- Finding when the maximum revenue occurs: 1 pt.
- If (c) was not correct, you can't get any credit.

(4) A survey of 100 households at a village produced the following result: 52 have Playstation 4, 49 have X-Box one, and 27 have neither of Playstation 4 nor X-Box one.

(a) (3 pts) How many have at least one of Playstation 4 or X-Box one?

Suppose that P is the set of households having Playstation 4, X is the set of households having X-box one. Then $n(P) = 52$, $n(X) = 49$ and $n(P' \cap X') = 27$.



$n(P \cup X) = 100 - 27 = 73$.

(b) (3 pts) How many have both of them?

$$n(P \cup X) = n(P) + n(X) - n(P \cap X) \Rightarrow 73 = 52 + 49 - n(P \cap X)$$
$$\Rightarrow z = n(P \cap X) = 28$$

(c) (3 pts) How many have a Playstation 4 but not X-Box one?

$$n(P) = x + z \Rightarrow 52 = x + 28 \Rightarrow x = 24$$

(5) Suppose that a single playing card is drawn at random from a standard card deck of 52 cards.

(a) (2 pts) Find the probability that the drawn card is a heart.

Suppose that H is the event that the chosen card is a heart. Then 13 cards among 52 cards are heart. So the probability is $P(H) = \frac{13}{52} = \frac{1}{4}$.

(b) (2 pts) Find the probability that the drawn card is a face card.

Suppose that F is the event that the chosen card is a face card. There are 12 face cards. Therefore the probability is $P(F) = \frac{12}{52} = \frac{3}{13}$.

(c) (3 pts) Find the probability that the drawn card is a heart or a face card.

Because there are 3 heart face cards, $P(H \cap F) = \frac{3}{52}$.

$$P(H \cup F) = P(H) + P(F) - P(H \cap F) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

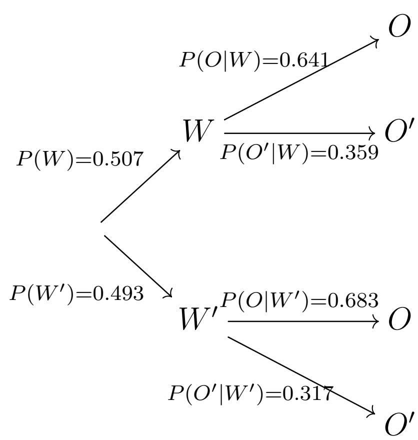
(d) (3 pts) Find the probability that the drawn card is not a heart.

$$P(H') = 1 - P(H) = 1 - \frac{1}{4} = \frac{3}{4}$$

(6) According to a recent report, 68.3% of men and 64.1% of women in the United States were overweight. It is also known that 49.3% of Americans are men and 50.7% are women.

(a) (3 pts) Find the probability that a randomly selected American is an overweight woman.

Suppose that O is the event that the chosen American is overweight, and W is the event that the chosen American is a woman.



$$P(O \cap W) = P(W)P(O|W) = 0.507 \cdot 0.641 \approx 0.3250$$

- Sketching the tree diagram with appropriate probabilities: 2 pts.
- Getting the answer: 3 pts.

(b) (5 pts) Find the probability that a randomly selected American is overweight.

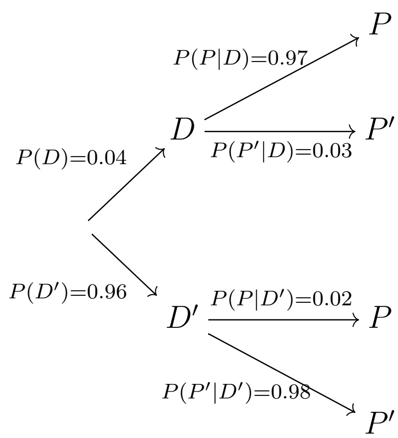
$$P(O) = P(W \cap O) + P(W' \cap O) = P(W)P(O|W) + P(W')P(O|W')$$
$$= 0.507 \cdot 0.641 + 0.493 \cdot 0.683 \approx 0.6617$$

- Stating $P(O) = P(W \cap O) + P(W' \cap O)$: 2 pts.

(7) A drug-screening test is used in a group of professional baseball players of whom 4% actually use illegal drugs. It is found that the test indicates positive in 97% of those who use drugs and 2% of those who do not.

(a) (4 pts) Find the probability that the test indicates positive for a randomly chosen player in the group.

Suppose that D is the event that the chosen player actually uses illegal drugs, and P is the event that the test is positive.



$$P(P) = P(P|D) + P(P|D') = P(D)P(P|D) + P(D')P(P|D')$$
$$= 0.04 \cdot 0.97 + 0.96 \cdot 0.02 = 0.058$$

- Sketching the tree diagram with appropriate probabilities: 2 pts.
- Getting the answer 0.058: 4 pts.

(b) (5 pts) What is the probability that a randomly chosen player in the group with positive test result actually uses drugs?

$$P(D|P) = \frac{P(D \cap P)}{P(P)} = \frac{0.04 \cdot 0.97}{0.058} \approx 0.6690$$

- Describing the probability $P(D|P)$ as a ratio of ordinary probabilities $\frac{P(D \cap P)}{P(P)}$: 2 pts.
- Getting the answer 0.6690: 5 pts.