# Computer project 3 Solution

This project must be submitted on paper. Keep in mind that the output should be a research report - you should provide answers and (mathematical/experimental) evidences. Feel free to use MS words, Pages, LaTeX, even handwriting, but include supporting evidences such as your sage codes, graphs and data. For the below problems, you *don't* need to find the precise answer unless there is a special indication. By using a computer algebra system, find the best solution you can find. The due date is April 30th.

Suppose that in Yellowstone National Park, the population of Elks and wolves are described by the following non-homogeneous prey-predator population model

$$\begin{aligned} P_{n+1} &= 1.04P_n - 0.15Q_n - 1.5 \\ Q_{n+1} &= 0.11P_n + 1.02Q_n - 15.6, \end{aligned}$$

where $P_n$ is the population of Elks in $n$-th year, and $Q_n$ is that of wolves (the unit of the population is hundred). If the population (of one species) becomes 0 or negative, then we will consider that the species becomes extinct.

The goal of this project is to find initial conditions $(P_0, Q_0)$ for which one of two species are extinct.

1. Find the fixed population $(p, q)$.

   ```
   x,y = var('x,y')
   solve([x == 1.04*x - 0.15*y -1.5, y== 0.11*x + 1.02*y - 15.6],x,y)

   [[x == (23700/173), y == (4590/173)]]
   ```

   Therefore
   $$(p, q) = \left(\frac{23700}{173}, \frac{4590}{173}\right) \approx (136.994, 26.532)$$
   is the fixed population.

2. Determine the stability of the fixed point.

   $$A = \begin{bmatrix} 1.04 & -0.15 \\ 0.11 & 1.02 \end{bmatrix}$$

```
A = matrix(QQ,[[1.04,-0.15],[0.11,1.02]])
ev = A.eigenvalues()
print "eigenvalues:", ev
print "the modulus of an eigenvalue:", ev[0].abs()
```

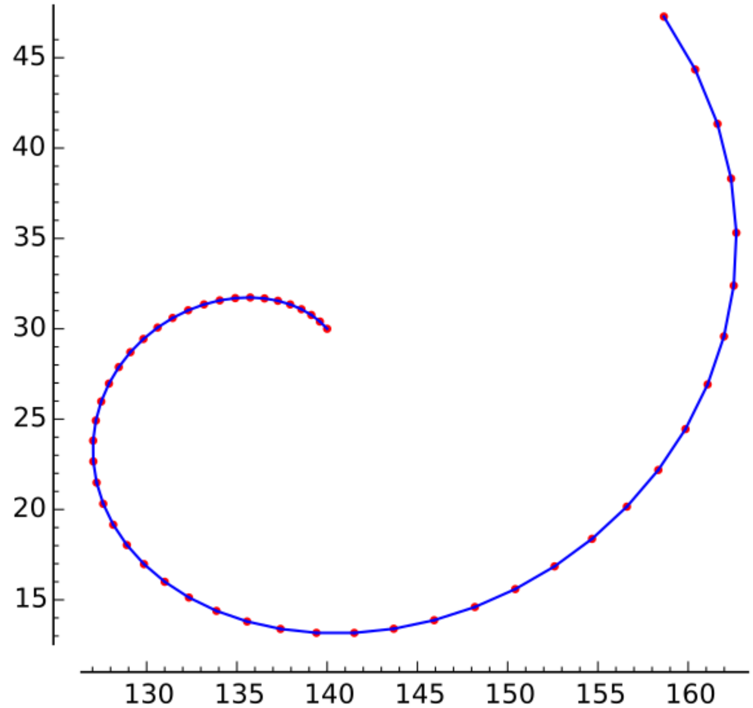
```
eigenvalues: [1.030000000000000? - 0.1280624847486570?*I,
1.030000000000000? + 0.1280624847486570?*I]
the modulus of an eigenvalue: 1.037930633520372?
```

Because the modulus of an eigenvalue is larger than one, the fixed point is a source.

For problems 1 and 2, you may compute without using computer algebra system.

3. For the initial condition $(P_0, Q_0) = (140, 30)$, sketch the phase graph for the first 50 iterations.

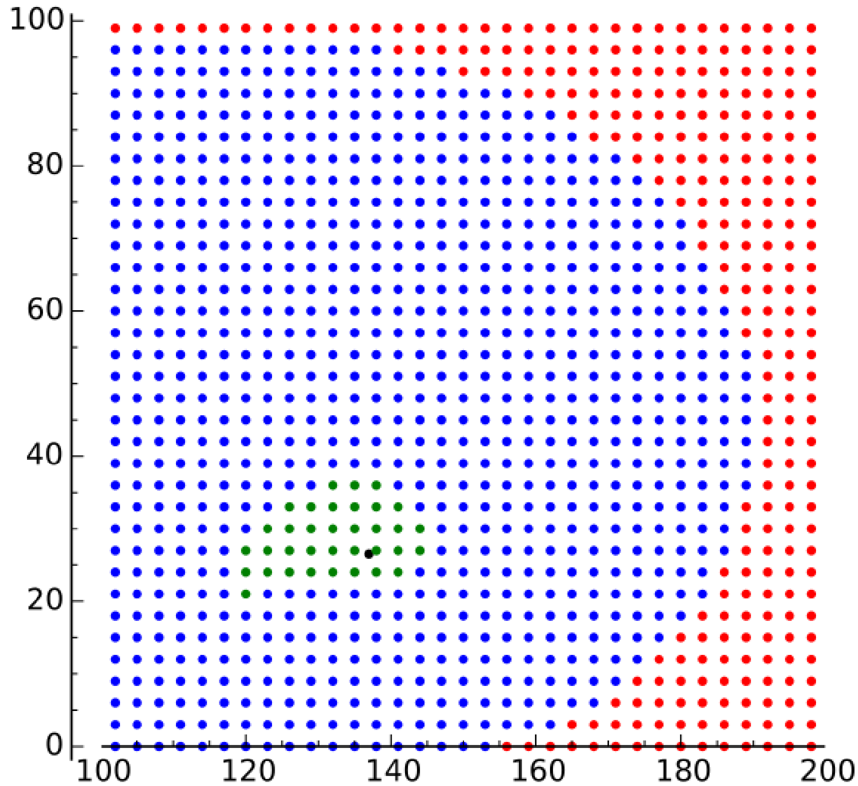
```
x = [140]
y = [30]
p = point([x[0],y[0]],rgbcolor='red',aspect_ratio=1)
for i in range(0,50):
    x.append(1.04*x[i]-0.15*y[i]-1.5)
    y.append(0.11*x[i]+1.02*y[i]-15.6)
    p = p + point([x[i+1],y[i+1]],rgbcolor='red')
    p = p + line([[x[i],y[i]],[x[i+1],y[i+1]]])
p.plot()
```




4. Take initial conditions $(P_0, Q_0)$ where $100 \leq P_0 \leq 200$, $0 \leq Q_0 \leq 100$ and $P_0, Q_0$ are integers which are multiples of 3. Plot a graph as the following: Plot the black fixed point. For each initial condition $(P_0, Q_0)$, if Elks become extinct in 50 iterations, plot a red point at $(P_0, Q_0)$. If wolves become extinct in 50 iterations, plot a blue point at $(P_0, Q_0)$. If no extinction occurs, plot a green point at $(P_0, Q_0)$. Product a colored image and attach it at the end of the paper.

```
fixed_point = [23700/173,4590/173]
P = point(fixed_point,rgbcolor='black',aspect_ratio=1)
for i in range(102,200,3):
    for j in range(0,100,3):
        x = [i]
        y = [j]
        point_color = 'green'
        for n in range(50):
            x.append(1.04*x[n]-0.15*y[n]-1.5)
            y.append(0.11*x[n]+1.02*y[n]-15.6)
            if x[n+1] <= 0:
                point_color = 'red'
                break
            if y[n+1] <= 0:
```

```
                point_color = 'blue'
                break
        P = P + point([i,j],rgbcolor=point_color)
P.plot()
```




- Each question is 6 points, and the overall completeness including formatting is 6 points. Remember that the output must be a summarized report, not a bulk of codes or answers.
- If you provide a complete calculation or appropriate reasoning including Sage code, you get 6 points. If the reasoning is incomplete, then you can get 2 or 4 points.
- Unless you provide computer codes and the output such as time series graph obtained by running the code, you cannot get the credit.